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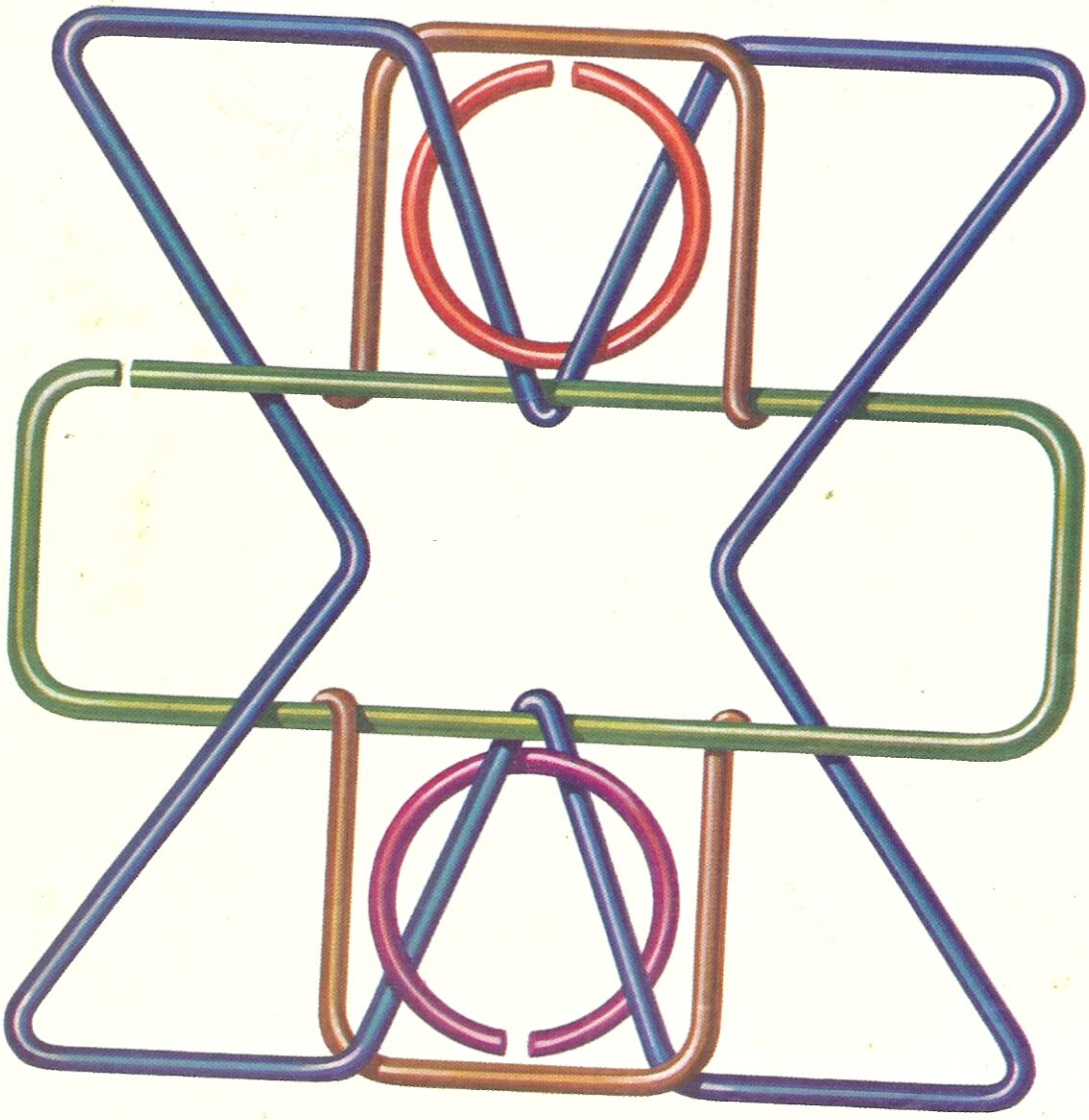

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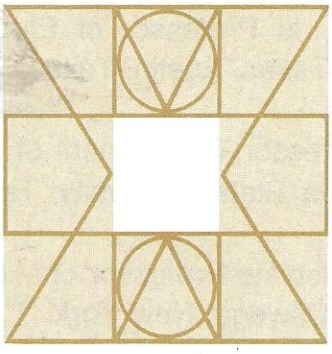

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1 SETS AND NUMERATION

SETS

{Presidents whose heads are carved on Mount Rushmore}

Description of set

{Washington, Jefferson, Lincoln, Theodore Roosevelt}

Listing the members

1. List the members.

a. {Great Lakes}

b. {the first 5 odd numbers}

2. Let $A = \{2, 4, 6, 8, 10\}$.

a. Is 6 a **member**, or **element**, of A? We write:

$6 \in A$

read: is a member of

b. Is 5 a member of A? We write:

$5 \notin A$

read: is not a member of

3. Let $V = \{a, e, i, o, u\}$ and $N = \{1, 2, 3, 4, 5, \ldots\}$.

a. How many members does V have? V is a **finite** set.

b. How many members does N have? N is an **infinite** set.

4. Let $A = \{1, 2, 3\}$, $B = \{1, 3, 5\}$, and $C = \{2, 3, 1\}$.

a. Do sets A and C contain the same members? Set A is equal to set C. We write: $A = C$.

b. Do sets A and B contain the same members? Set A is not equal to set B. We write: $A \neq B$.

EXERCISES

List the members.

1. C = {the first three presidents of the United States}

2. E = {natural numbers less than 10}

3. F = {natural numbers greater than 10}

4. G = {odd numbers less than 20}

5. H = {even numbers greater than 100}

6. I = {the least natural number}

7. J = {digits in our numeration system}

8. P = {odd numbers greater than 0 and less than 10}

Let $A = \{0, 1, 2\}$, $B = \{2, 6, 4, 0\}$, $C = \{1, 3, 5, 7\}$, and $D = \{0, 2, 4, 6\}$. True or false?

9. $2 \in A$ **10.** $2 \in C$ **11.** $3 \notin A$ **12.** $3 \notin C$

13. $9 \in A$ **14.** $A \neq B$ **15.** $B = D$ **16.** $C \neq D$

Which are finite sets? Which are infinite sets?

17. A = {all cars in the U.S.} **18.** E = {all even numbers}

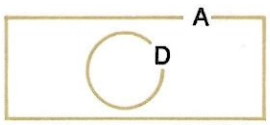
★ **19.** P = {products of zero and the natural numbers}

★ **20.** B = {all books in an empty bookcase}

SUBSETS

Let $A = \{\text{all animals}\}$ and $D = \{\text{all dogs}\}$.

This **Venn diagram** shows that each member of D is a member of A. D is a **subset** of A. We write: $D \subseteq A$.



1. Let $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $A = \{1, 3, 5, 7\}$. Why is $A \subseteq B$ a true statement?

2. Bill can have as many as three desserts from this set: $D = \{\text{cake, ice cream, pie}\}$. Let's find out what Bill might choose.

   a. He may choose all three desserts. List the members of this set.

   b. He may choose two desserts. Find three subsets A, B, and C, each with two members.

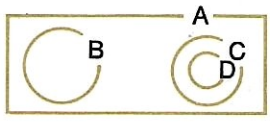
   c. He may choose one dessert. Find three subsets E, F, and G, each with one member.

   d. Might he choose no dessert? This subset is the **empty set,** or the **null** set. We write ϕ.

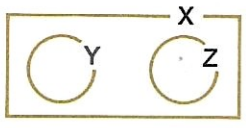
3. Find four subsets of $H = \{1, 2\}$.

4. What does $E \subseteq D$ mean? What does $D \subseteq E$ mean?

5. This Venn diagram shows that $B \subseteq A$, $C \subseteq A$, and $D \subseteq C$. What do these Venn diagrams show?

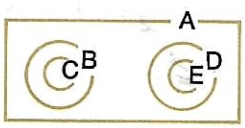


a.



b.

c.



6. Let $A = \{\text{Ann, Bill, Peg}\}$, $B = \{\text{Ann, Peg}\}$, and $C = \{\text{Peg}\}$. Draw a Venn diagram to show how the sets are related.

EXERCISES

Let $A = \{2, 4\}$, $B = \{2\}$, $C = \{1, 7\}$, and $E = \{2, 5, 8\}$. True or false?

1. $B \subseteq A$ **2.** $A \subseteq B$ **3.** $C \subseteq E$ **4.** $E \subseteq E$

5. $\phi \subseteq E$ **6.** $\phi \subseteq B$ **7.** $B \subseteq B$ **8.** $A \subseteq A$

Let $X = \{\text{Ted, Rose, Pete, Ann}\}$.

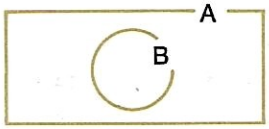
9. Find 16 subsets of X.

Give the meanings.

10. $A \subseteq B$ **11.** $B \subseteq A$ **12.** $F \subseteq F$ **13.** $\phi \subseteq M$

What do these Venn diagrams show?

14.



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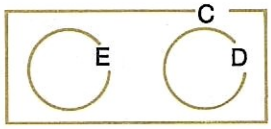
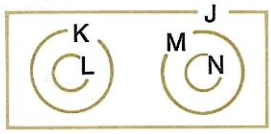


16.

17.



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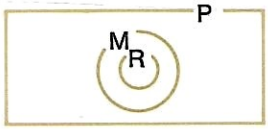

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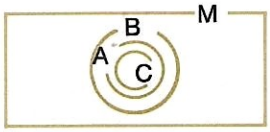


Draw Venn diagrams.

20. $B \subseteq D$ **21.** $A \subseteq B$, $C \subseteq B$, and $R \subseteq C$

Let $A = \{2, 3, 6, 8\}$, $B = \{2, 3, 6\}$, $C = \{2\}$, and $D = \{8\}$.

22. Draw a Venn diagram to show how the sets are related.

Let $N = \{1, 2, 3, 4, \ldots\}$ and $O = \{1, 3, 5, 7, \ldots\}$.

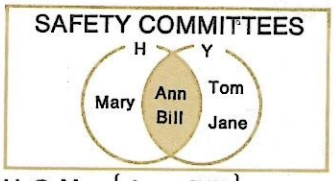
★**23.** True or false? $O \subseteq N$.

INTERSECTION OF SETS

Hall: $H = \{\text{Mary, Ann, Bill}\}$

Yard: $Y = \{\text{Tom, Ann, Bill, Jane}\}$

The **intersection** of H and Y is the set that contains only the common members of H and Y.



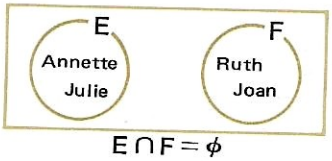
$H \cap Y = \{\text{Ann, Bill}\}$

↑ symbol for intersection

1. Let $A = \{1, 4, 8\}$, $B = \{4, 7, 8, 10\}$, $C = \{1, 6, 9, 10\}$, and $D = \{1, 6, 9\}$. Find.

a. $A \cap B$ **b.** $B \cap C$ **c.** $C \cap D$ **d.** $A \cap D$

2. Sets E and F have no members in common. Their intersection is the empty set. E and F are called **disjoint sets**.



$E \cap F = \phi$

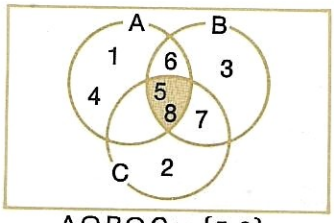
Which pairs of sets are disjoint? Which are not?

a. $A = \{\text{Sue, Rose, Amy}\}$ $B = \{\text{Ann, Ida, Mae}\}$

b. $C = \{1, 2, 3, 4\}$ $D = \{5, 3, 6, 8\}$

c. $R = \{\text{Joe, 8, Dave}\}$ $T = \{\text{Ruby, Dave, 8}\}$

3. Sometimes three sets have common members. The shaded portion shows the intersection of A, B, and C.



$A \cap B \cap C = \{5, 8\}$

Let $D = \{1, 2, 3, 4\}$, $E = \{3, 4, 5, 6, 7\}$, $F = \{4, 5, 8, 9\}$, and $G = \{2, 10, 12\}$. Find.

a. $D \cap G$ **b.** $D \cap E \cap F$ **c.** $E \cap F \cap G$ **d.** $D \cap F \cap G$

4. Let $A = \{\text{b, a, r, t}\}$, $B = \{\text{b, a, s, e}\}$, $C = \{\text{a, i, m}\}$, and $D = \{\text{i, t}\}$. Draw Venn diagrams.

a. $A \cap B$ **b.** $B \cap D$ **c.** $A \cap B \cap C$ **d.** $A \cap C \cap D$

EXERCISES

Find the intersections.

1. $A = \{a, b, o, v, e\}$ and $B = \{b, e, l, o, w\}$

2. $M = \{\text{Mary, Jane}\}$ and $D = \{\text{Dick}\}$

3. $X = \{1, 6, 9, 10\}$ and $Y = \{3, 6, 9\}$

4. $R = \{1, 2, 3\}$ and $S = \{3\}$

Let $L = \{1, 2, 3, 4, 5\}$, $M = \{4, 5, 6\}$, $N = \{4, 5\}$ and $P = \{7, 8, 9\}$. Find.

5. $L \cap M$ 6. $L \cap N$ 7. $L \cap P$ 8. $M \cap N$

9. $M \cap P$ 10. $N \cap P$ 11. $L \cap M \cap N$ 12. $L \cap M \cap P$

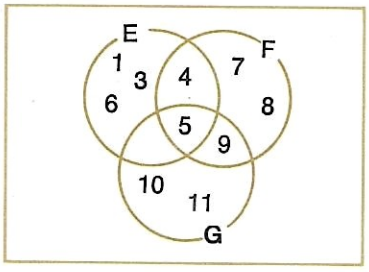
Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{7, 8, 9\}$, and $D = \{0, 6, 7\}$. True or false?

13. $A \cap B = \{1\}$ 14. $A \cap C = \phi$ 15. $C \cap D = \{7\}$

16. $A \cap B = B$ 17. $A \cap A = A$ 18. $A \cap \phi = A$

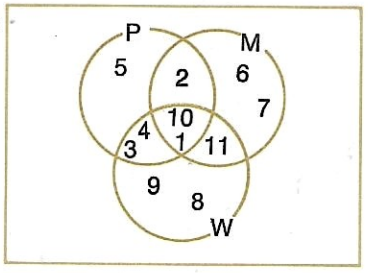
Study the Venn diagrams. Complete.

19.



E∩F∩G = _____

20.



P∩M∩W = _____

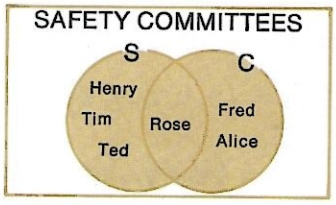
Let $R = \{m, a, t, h\}$, $S = \{h, a, r, e\}$, $T = \{r, e, a, l\}$ and $X = \{b, i, t\}$. Draw Venn diagrams.

21. $R \cap S$ 22. $S \cap X$ 23. $R \cap S \cap T$ 24. $R \cap T \cap X$

UNION OF SETS

Stairs: $S = \{\text{Henry, Tim, Ted, Rose}\}$
Cafeteria: $C = \{\text{Rose, Fred, Alice}\}$

The **union** of sets S and C is the set that contains all the members that are in S, or in C, or in both S and C.



So, $S \cup C = \{\text{Henry, Tim, Ted, Rose, Fred, Alice}\}$.

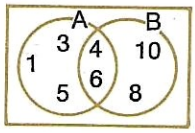
$\cup$ — symbol for union

1. Let $A = \{2, 4, 6\}$, $B = \{2, 4\}$, $C = \{1, 3, 5\}$, and $D = \{5, 10\}$. Find.

a. $A \cup C$ **b.** $C \cup D$ **c.** $A \cup B$ **d.** $A \cup C \cup D$

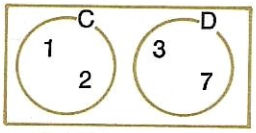
2. Study the Venn diagrams. Complete.

a.



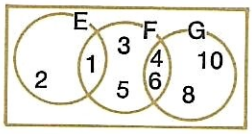
A U B = ____

b.



C U D = ____

c.



E U F U G = ____

EXERCISES

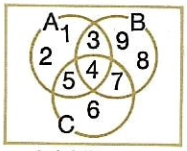
Let $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 6, 8\}$, $C = \{1, 2\}$, and $D = \{1, 2, 3, 4\}$. Find.

1. $A \cup B$ **2.** $A \cup C$ **3.** $A \cup D$ **4.** $D \cup C$

5. $B \cup D$ **6.** $C \cup D$ **7.** $A \cup C \cup D$ **8.** $A \cup B \cup D$

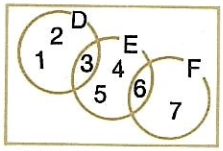
Study the Venn diagrams. Complete.

9.



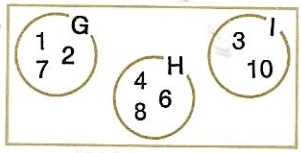
A U B = ____
A U B U C = ____

10.



D U E = ____
D U E U F = ____

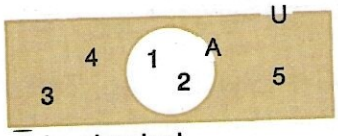
11.



G U I = ____
G U H U I = ____

UNIVERSAL AND COMPLEMENTARY SETS

$U = \{1, 2, 3, 4, 5\}$ and $A = \{1, 2\}$. U is called the **universal set**, or **universe**. The set of members *not* in A is $\{3, 4, 5\}$. This set is called the **complement** of set A.



$\bar{A}$ is shaded.

read: the complement of set A

1. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1\}$, $B = \{2, 4, 6\}$, and $C = \{9, 10\}$. Find.

a. $\bar{A}$ **b.** $\bar{B}$ **c.** $\bar{C}$ **d.** $\bar{U}$ **e.** $A \cap \bar{A}$ **f.** $A \cup \bar{A}$

2. To show that U has 100 members, we write:

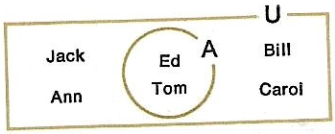
$n(U) = 100$

read: the number of members in U

Give the meanings.

a. $n(U) = 200$ **b.** $n(R) = 140$ **c.** $n(T) = 60$

3. Study the diagram. Find.



a. $n(U)$ **b.** $n(A)$ **c.** $n(\bar{A})$

EXERCISES

Let $U = \{a, b, c, d, e, f, g, h\}$, $A = \{a, b, e, f\}$, $B = \{c, a, h\}$ and $C = \{h\}$. Find.

1. $\bar{A}$ **2.** $\bar{B}$ **3.** $\bar{C}$ **4.** $n(U)$ **5.** $n(A)$

6. $n(B)$ **7.** $n(\bar{A})$ **8.** $n(\bar{B})$ **9.** $n(\bar{C})$ **10.** $B \cap \bar{B}$

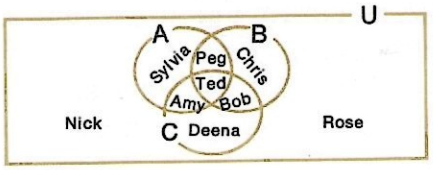
Consider U and two of its disjoint subsets A and B. Let $n(U) = 119$, $n(A) = 48$, and $n(B) = 25$. Find.

11. $n(\bar{A})$ **12.** $n(\bar{B})$ ★**13.** $n(\bar{A} \cap \bar{B})$ ★**14.** $n(\bar{A} \cup \bar{B})$

UNION AND INTERSECTION

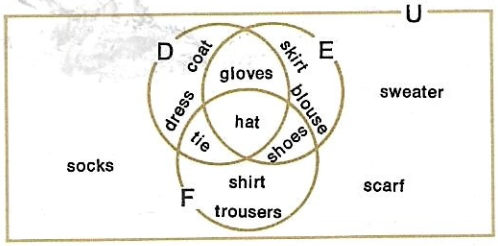
U = {Sylvia, Peg, Ted, Amy, Deena, Bob, Chris, Rose, Nick}

A = {Sylvia, Peg, Ted, Amy}
B = {Peg, Ted, Bob, Chris}
C = {Amy, Ted, Bob, Deena}



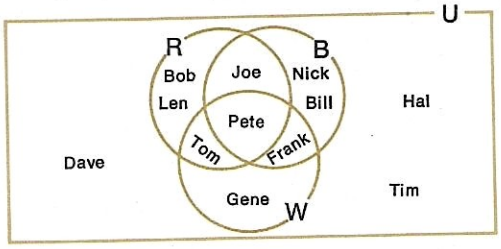
$A \cap B \cap C$ = {Ted}
$A \cup B \cup C$ = {Slyvia, Peg, Ted, Amy, Deena, Bob, Chris}

1. Study the diagram. List the members.



a. D **b.** E **c.** F **d.** $D \cap E$ **e.** $D \cap E \cap F$
f. $\overline{F}$ **g.** $D \cup E$ **h.** $E \cap F$ **i.** $D \cup F$ **j.** $D \cup E \cup F$

2. Let R = {boys wearing red shirts}, B = {boys wearing blue ties}, and W = {boys wearing white jackets}.



a. List the members of R. of B. of W.

b. What is Joe wearing? Frank? Tom? Pete?

c. Which boys are wearing blue ties but are wearing neither red shirts nor white jackets?

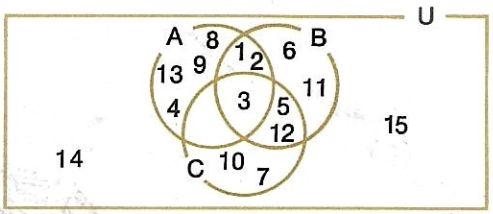
3. Let $L = \{a, b, c, d, e\}$, $M = \{c, d, e, f, g, h\}$, and $N = \{d, h, m\}$.

a. Draw a Venn diagram to show how the sets are related.

b. What is $L \cap M \cap N$? **c.** What is $L \cup M \cup N$

EXERCISES

Study the diagram. List the members.



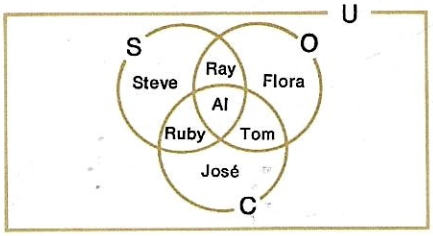
1. U **2.** A **3.** B **4.** C

5. $A \cap B$ **6.** $A \cup B$ **7.** $B \cap C$ **8.** $B \cup C$

9. $A \cap C$ **10.** $A \cup C$ **11.** $\overline{A}$ **12.** $\overline{B}$

13. $\overline{C}$ **14.** $A \cap B \cap C$ **15.** $A \cup B \cup C$ **16.** $\overline{A} \cap \overline{C}$

$S = \{\text{students on safety patrol}\}$
$O = \{\text{class officers}\}$
$C = \{\text{students on cafeteria patrol}\}$
Steve belongs only to the safety patrol. To what organizations do these students belong?



17. Ruby **18.** José **19.** Tom **20.** Al **21.** Flora **22.** Ray

Let $M = \{\text{Alice, Mary, Tom, Bill}\}$, $N = \{\text{Tom, Mary, Fred, Dick}\}$, and $O = \{\text{Ann, Mary, Shirley, Carolyn, Sam}\}$.

23. Draw a Venn diagram to show how the sets are related.

24. What is $M \cap N \cap O$? **25.** What is $M \cup N \cup O$?

SOLVING PROBLEMS USING SETS

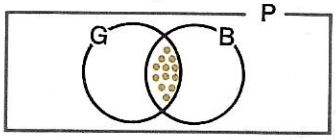
Consider this problem.

28 girls.
20 of them have long hair.
16 of them have curly hair.
12 of them have both long and curly hair.

How many girls have long hair, but not curly hair?
How many girls have curly hair, but not long hair?
How many girls have neither long hair nor curly hair?

To answer these questions, we shall use a Venn diagram.

Let $P = \{\text{girls}\}$, so $n(P) = 28$. $G = \{\text{girls having long hair}\}$, so $n(G) = 20$. $B = \{\text{girls having curly hair}\}$, so $n(B) = 16$ and $n(G \cap B) = 12$.

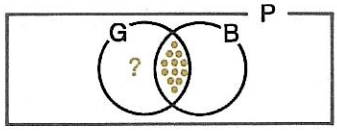


Each dot represents a girl.

1. We know that $n(G) = 20$ and that $n(G \cap B) = 12$.

a. How many are in G, but not in $G \cap B$?

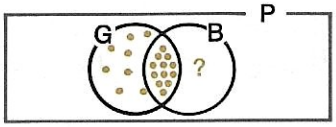
b. How many have long hair, but not curly hair?



2. We know that $n(B) = 16$ and that $n(G \cap B) = 12$.

a. How many are in B, but not in $G \cap B$?

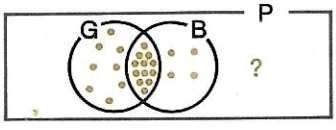
b. How many have curly hair, but not long hair?



3. We know that $n(P) = 28$.

a. What is $n(G \cup B)$?

b. How many have neither long hair nor curly hair?



EXERCISES

Solve these problems.

1. 30 students in a class.
20 of them like football.
17 of them like baseball.
8 of them like both football and baseball.

How many like football, but not baseball?

How many like baseball, but not football?

How many like neither football nor baseball?

2. 38 boy scouts on a hiking trip.
32 had breakfast.
35 had lunch.
30 had both breakfast and lunch.

How many had only breakfast?

How many had only lunch?

How many had neither lunch nor breakfast?

★**3.** 300 students in grade 8.
240 voted for school president.
220 voted for school vice president.
200 voted for both school president and vice president.

How many voted only for school president?

How many voted only for school vice president?

How many did not vote for either school president or school vice president?

BRAINTEASER

Ann is 13 years old and has a brother who is one year old. Ann is now 13 times as old as her brother. Next year, she will be 7 times as old as her brother. Two years from now, she will be 5 times as old as her brother. In how many years will she be twice as old as her brother?

CODES

Code:

W	A	S	H	I	N	G	T	O	D
0	1	2	3	4	5	6	7	8	9

The cost of the car is $2,900.

1. Code:

F	R	I	D	A	Y	H	O	M	E
0	1	2	3	4	5	6	7	8	9

What numbers are represented?

a. DF **b.** HY **c.** HEF **d.** AYYI **e.** OOHDIF

2. Use the code. Represent these numbers.

a. 21 **b.** 12 **c.** 364 **d.** 4,002 **e.** 34,625

3. Make up a code of your own for representing numbers. Represent the numbers from 257 to 262 in your code.

EXERCISES

Code:

N	U	M	B	E	R	T	A	I	L
0	1	2	3	4	5	6	7	8	9

What numbers are represented?

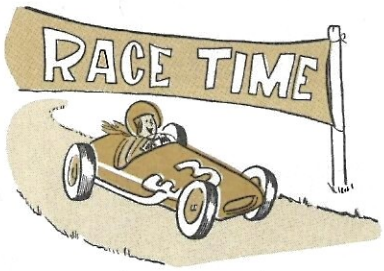
1. ME **2.** TEAR **3.** MAIL **4.** BE

5. AMBER **6.** TUMBLE **7.** UMBER **8.** TAIL

Represent these numbers.

9. 56 **10.** 65 **11.** 104 **12.** 497

13. 3,716 **14.** 3,167 **15.** 7,356 **16.** 4,000



See how fast you can add without making errors.

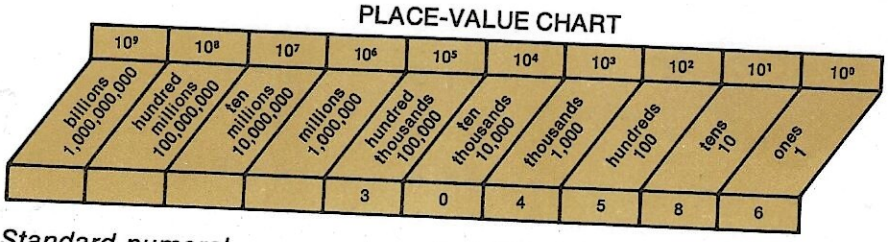
		$4+6$	$5+5$	$3+2$
		$0+9$	$7+6$	$8+9$
		$3+5$	$8+8$	$8+2$
		$7+9$	$3+7$	$7+7$
		$4+4$	$4+0$	$9+5$
		$2+9$	$9+7$	$1+7$
$9+9$	$2+8$	$5+6$	$2+2$	$6+5$
$4+1$	$5+4$	$0+5$	$3+9$	$3+4$
$0+6$	$1+3$	$1+4$	$8+3$	$6+6$
$7+2$	$2+6$	$9+7$	$7+1$	$8+7$
$2+5$	$1+2$	$8+1$	$8+9$	$6+1$
$5+4$	$3+3$	$5+7$	$1+6$	$4+8$
$6+3$	$1+9$	$1+5$	$8+6$	$9+4$
$4+0$	$2+3$	$2+1$	$6+9$	$2+4$
$3+1$	$9+8$	$5+9$	$7+4$	$5+0$

See how fast you can subtract without making errors.

$9-3$	$14-3$	$8-0$	$15-1$	$12-2$
$12-5$	$11-4$	$9-7$	$12-7$	$18-3$
$13-7$	$17-9$	$14-7$	$16-9$	$14-6$
$11-2$	$10-0$	$13-3$	$16-5$	$9-6$
$17-7$	$11-5$	$9-4$	$13-8$	$11-3$
$12-8$	$18-5$	$13-6$	$15-3$	$17-8$
$2-1$	$16-3$	$15-7$	$16-6$	$18-0$
$11-1$	$13-2$	$17-6$	$14-4$	$12-9$
$13-4$	$7-5$	$18-9$	$16-8$	$18-5$
$15-6$	$18-4$	$14-5$	$15-9$	$11-9$
$17-5$	$16-7$	$13-9$	$17-4$	$14-8$
$14-9$	$15-8$	$8-3$	$18-8$	$13-5$
$17-3$	$18-7$	$12-4$	$11-7$	$15-4$

HINDU-ARABIC SYSTEM OF NUMERATION

PLACE-VALUE CHART



Standard numeral ↓ 304,586 = $3 \cdot 10^5 + 0 \cdot 10^4 + 4 \cdot 10^3 + 5 \cdot 10^2 + 8 \cdot 10^1 + 6 \cdot 10^0$ ← *Expanded numeral*

1. Write in exponential form.

a. $10 \cdot 10$ **b.** $10 \cdot 10 \cdot 10$ **c.** $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

2. Complete for the numeral 3,462. Use a power of 10.

a. The digit 3 has the value $3 \cdot$ ____.

b. The digit 4 has the value $4 \cdot$ ____.

c. The digit 6 has the value $6 \cdot$ ____.

d. The digit 2 has the value $2 \cdot$ ____.

3. Write expanded numerals. Use exponents.

a. 157 **b.** 24,347 **c.** 2,436,871

4. The value of the digit 7 in 2,756 is $7 \cdot 10^2$, or 700. Find the values of the underlined digits.

a. 4,5<u>8</u>3 **b.** 23,<u>9</u>02 **c.** 46<u>7</u>,<u>7</u>34

5. Write standard numerals.

a. $5 \cdot 10^6 + 7 \cdot 10^5 + 6 \cdot 10^4 + 3 \cdot 10^3 + 0 \cdot 10^2 + 0 \cdot 10^1 + 0 \cdot 10^0$

b. $5 \cdot 10^6 + 5 \cdot 10^5 + 7 \cdot 10^4 + 6 \cdot 10^3 + 5 \cdot 10^2 + 2 \cdot 10^1 + 3 \cdot 10^0$

EXERCISES

Write in exponential form.

1. $10 \cdot 10 \cdot 10 \cdot 10$

2. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

3. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

4. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

Write expanded numerals. Use exponents.

5. 237 **6.** 409 **7.** 3,469

8. 7,034 **9.** 11,023 **10.** 17,409

11. 346,012 **12.** 840,000 **13.** 7,834,643

Find the values of the underlined digits.

14. $\underline{2}14$ **15.** $\underline{1}24$ **16.** $\underline{5}{,}629$

17. $6{,}\underline{5}29$ **18.** $\underline{7}4{,}384$ **19.** $4\underline{7}{,}384$

20. $\underline{5}6{,}019$ **21.** $60{,}\underline{5}19$ **22.** $24{,}3\underline{8}8$

23. $35{,}\underline{8}84$ **24.** $61\underline{3}{,}\underline{3}19$ **25.** $\underline{33}1{,}619$

26. $4{,}32\underline{4}{,}162$ **27.** $7{,}8\underline{4}1{,}312$ **28.** $74{,}\underline{8}93{,}\underline{8}12$

Write standard numerals.

29. $4 \cdot 10^2 + 3 \cdot 10^1 + 2 \cdot 10^0$

30. $8 \cdot 10^3 + 7 \cdot 10^2 + 6 \cdot 10^1 + 3 \cdot 10^0$

31. $9 \cdot 10^4 + 8 \cdot 10^3 + 7 \cdot 10^2 + 6 \cdot 10^1 + 4 \cdot 10^0$

32. $8 \cdot 10^6 + 7 \cdot 10^5 + 0 \cdot 10^4 + 0 \cdot 10^3 + 2 \cdot 10^2 + 1 \cdot 10^1 + 0 \cdot 10^0$

ROUNDING

Let's round 64,293 to the nearest ten thousand.

6 4,2 9 3

↑ The digit to the right of the ten-thousands place is less than 5.

So, 64,293 ≐ 60,000.

↑ read: is approximately equal to

Let's round 64,293 to the nearest hundred.

6 4 , 2 9 3

↑ The digit to the right of the hundreds place is 5 or more.

So, 64,293 ≐ 64,300.

1. Round to the nearest hundred.

   a. 563 **b.** 741 **c.** 3,457 **d.** 89

2. Round to the nearest thousand.

   a. 2,403 **b.** 6,934 **c.** 243,571 **d.** 656

3. Round to the nearest ten.

   a. 35 **b.** 42 **c.** 39 **d.** 37

4. Round to the nearest dollar.

   a. $5.98 **b.** $27.49 **c.** $346.47 **d.** $829.50

5. Round to the nearest hundred dollars. Subtract.

   a. $567.42 − $243.00 **b.** $7,346.43 − $2,853.29

6. Round each to the nearest ten, hundred, and thousand.

   a. 567 **b.** 643 **c.** 612 **d.** 594

7. Round 3,456,023 to the nearest million.

8. $746 \doteq 700$ means 746 was rounded to the nearest hundred. To what place has each been rounded?

a. $967 \doteq 1{,}000$ **b.** $850{,}496 \doteq 850{,}500$ **c.** $456{,}444 \doteq 456{,}440$

EXERCISES

Round to the nearest hundred.

1. 349 **2.** 680 **3.** 1,824 **4.** 7,555

Round to the nearest thousand.

5. 6,145 **6.** 7,825 **7.** 16,390 **8.** 18,500

Round to the nearest million.

9. 3,416,999 **10.** 4,510,000 **11.** 256,812,700

Complete.

	Number	*Nearest Ten*	*Nearest Thousand*	*Nearest Million*	*Nearest Hundred*
12.	17,016,437				
13.	18,399,486				
14.	418,500,000				
15.	3,728,543				
16.	847,821				

Add. Round each sum to the nearest ten dollars.

17. $\$423 + \$746 + \$924$ **18.** $\$4{,}371 + \$89 + \$685$

Subtract. Round each difference to the nearest hundred dollars.

19. $\$8{,}764 - \$5{,}860$ **20.** $\$526{,}372 - \$39{,}821$

To what place has each been rounded?

21. $2{,}499 \doteq 2{,}000$ **22.** $13{,}568 \doteq 13{,}570$ **23.** $834{,}999 \doteq 830{,}000$

ROMAN SYSTEM OF NUMERATION

Roman	I	V	X	L	C	D	M
Hindu-Arabic	1	5	10	50	100	500	1,000

A subtraction principle is used in these six cases.

IV = 5 − 1, or 4
XL = 50 − 10, or 40
CD = 500 − 100, or 400
IX = 10 − 1, or 9
XC = 100 − 10, or 90
CM = 1,000 − 100, or 900

At all other times, an addition principle is used.

1. XXX means 10 + 10 + 10, or 30. What do these mean?

a. III **b.** CCC **c.** MM **d.** XX

2. **a.** Why isn't VV a correct name for 10?

b. Why isn't LL a correct name for 100?

c. Why isn't DD a correct name for 1,000?

3. $\overline{X}$ means 10 × 1,000, or 10,000. What do these mean?

a. $\overline{L}$ **b.** $\overline{C}$ **c.** $\overline{D}$ **d.** $\overline{M}$

4. The order of operations is subtract first, then add.

$$\begin{aligned} XIV &= 10 + (5 - 1) \\ &= 10 + 4, \text{ or } 14 \end{aligned}$$

What do these mean?

a. XIX **b.** MXC **c.** CXL **d.** MCM

5. Write Hindu-Arabic numerals.

a. XXIII **b.** MMCCC **c.** XVI **d.** CLX

e. XXIV **f.** XIX **g.** MCD **h.** $\overline{V}$

6. Let's write the Roman numeral for 29.

Think: $29 = 20 + 9$
$= XX + IX$, or XXIX

Write Roman numerals.

a. 3 **b.** 20 **c.** 35 **d.** 310

e. 9 **f.** 42 **g.** 14 **h.** 190

EXERCISES

1. How many symbols are used in the Roman system?

2. Is there a symbol for zero in the Roman system?

3. Does the Roman system use place value?

Write Hindu-Arabic numerals.

4. XX **5.** CXX **6.** CCXIII **7.** MMCCII

8. XVI **9.** XXVII **10.** LXVII **11.** MDCLXVI

12. XXIV **13.** XXIX **14.** XLV **15.** MCM

16. CCXC **17.** MCD **18.** $\overline{X}$MM **19.** $\overline{X}$MMIV

20. MCMLXXV **21.** MMI **22.** $\overline{MD}$ **23.** $\overline{MDVI}$

Write Roman numerals.

24. 22 **25.** 61 **26.** 210 **27.** 2,323

28. 18 **29.** 39 **30.** 404 **31.** 1,975

32. 959 **33.** 1,999 **34.** 1,300 **35.** 341

36. 111 **37.** 704 **38.** 437 **39.** 106

40. 809 **41.** 610 **42.** 2,563 **43.** 1,508,000

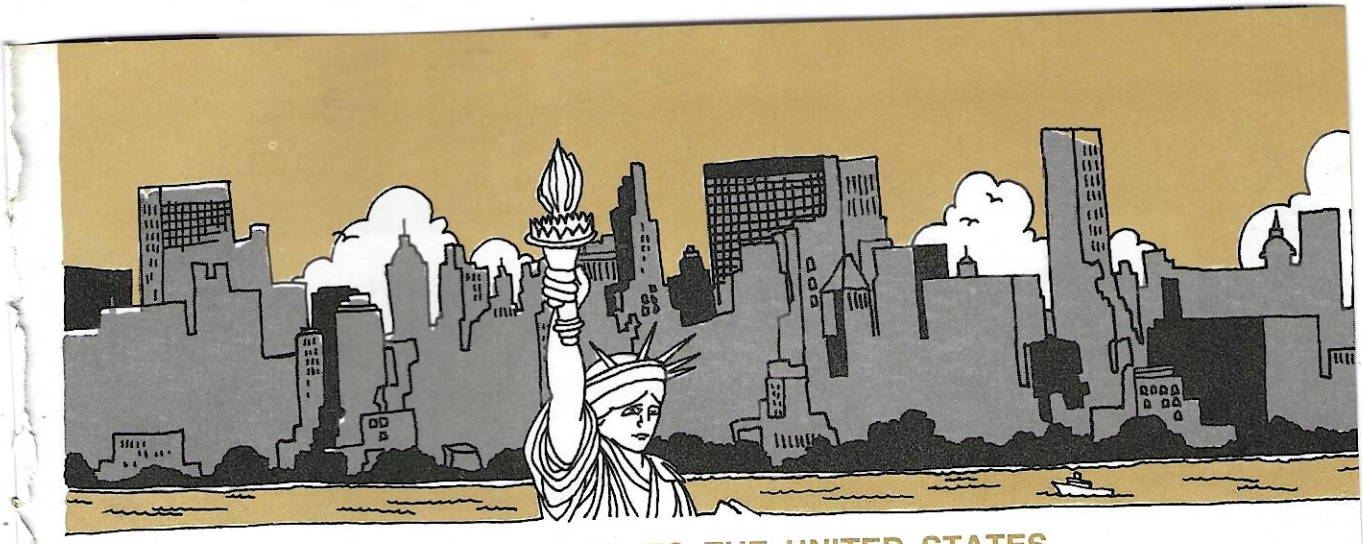
RACE TIME

See how fast you can multiply without making errors.

3 · 1	8 · 9	2 · 5	2 · 3	4 · 2
0 · 2	3 · 7	6 · 2	7 · 8	6 · 6
3 · 8	2 · 8	9 · 8	9 · 9	6 · 8
4 · 6	9 · 6	4 · 5	8 · 4	5 · 9
9 · 3	4 · 2	5 · 3	6 · 7	3 · 3
2 · 7	9 · 7	8 · 2	8 · 3	9 · 4
3 · 6	9 · 2	7 · 0	6 · 5	4 · 8
8 · 5	8 · 7	5 · 8	5 · 4	6 · 4
7 · 2	5 · 5	8 · 6		
3 · 5	4 · 7	6 · 3		
6 · 9	9 · 0	7 · 5		
7 · 7	8 · 8	7 · 3		
2 · 9	7 · 4	5 · 2		
8 · 0	5 · 7	7 · 6		

See how fast you can divide without making errors.

4 ÷ 2	45 ÷ 9	36 ÷ 9	3 ÷ 3	14 ÷ 7
18 ÷ 6	28 ÷ 4	14 ÷ 2	48 ÷ 6	0 ÷ 7
40 ÷ 5	60 ÷ 6	8 ÷ 8	2 ÷ 1	16 ÷ 8
36 ÷ 6	25 ÷ 5	9 ÷ 3	1 ÷ 1	42 ÷ 7
12 ÷ 6	72 ÷ 8	64 ÷ 8	45 ÷ 5	0 ÷ 8
56 ÷ 7	10 ÷ 2	20 ÷ 5	12 ÷ 2	28 ÷ 7
18 ÷ 3	0 ÷ 8	0 ÷ 5	21 ÷ 3	18 ÷ 1
6 ÷ 2	35 ÷ 1	30 ÷ 6	56 ÷ 8	10 ÷ 5
27 ÷ 3	12 ÷ 4	27 ÷ 9	36 ÷ 4	63 ÷ 7
63 ÷ 9	54 ÷ 9	21 ÷ 7	0 ÷ 6	16 ÷ 2
12 ÷ 3	8 ÷ 2	54 ÷ 6	28 ÷ 4	72 ÷ 9
6 ÷ 6	12 ÷ 3	49 ÷ 7	26 ÷ 1	15 ÷ 3
2 ÷ 2	24 ÷ 3	4 ÷ 4	24 ÷ 4	35 ÷ 5
0 ÷ 9	20 ÷ 4	18 ÷ 9	6 ÷ 3	30 ÷ 5
18 ÷ 2	24 ÷ 8	24 ÷ 6	15 ÷ 5	48 ÷ 8
40 ÷ 8	11 ÷ 1	0 ÷ 3	42 ÷ 6	16 ÷ 4



IMMIGRATION TO THE UNITED STATES

1. From 1820–1969, a period of 150 years, 44,789,312 immigrants came to the United States. Of these 35,803,436 came from European countries. How many did not come from European countries?

2. During the 150-year period, 6,906,645 immigrants came from Germany; 5,149,119 from Italy; 4,777,727 from England; 4,712,680 from Ireland; 3,941,858 from Canada; and 3,346,455 from Russia. How many came from these countries?

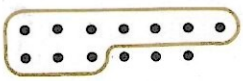
3. In 1790, the population of the U.S, was 3,929,213. The greatest number of immigrants, which was 8,795,000, came in the period 1901–1910. How many more immigrants was that than the total population in 1790?

4. In the period 1951–1960, immigrants of all ages came: 595,900 were under 16; 1,596,327 were 16–44; and 343,252 were 45 and over. In which age group was the greatest number of immigrants?

5. During the same 10-year period, 1,360,058 immigrants were women and 1,155,421 were men. How many more women than men came to the United States during 1951–1960?

6. Most of the immigrants found life in the United States trying. In one year, about 395,000 gave up and returned to their original homes. How many more or less than a half-million left the United States that year?

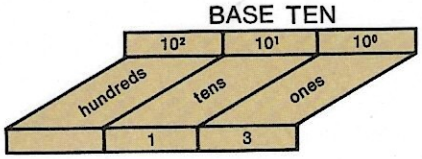
NUMERATION SYSTEMS: DIFFERENT BASES

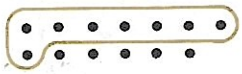


Grouped by tens

1 ten + 3 ones
13_{ten}

BASE TEN

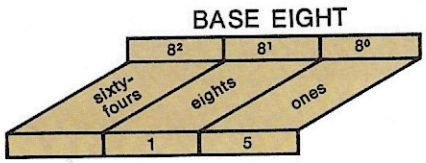




Grouped by eights

1 eight + 5 ones
15_{eight}

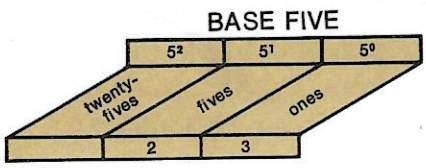
BASE EIGHT



Grouped by fives

2 fives + 3 ones
23_{five}

BASE FIVE



We see that $13_{\text{ten}} = 15_{\text{eight}} = 23_{\text{five}}$.

1. Let's find the base-ten numeral for 214_{five}. Complete.

$$214_{\text{five}} = 2 \cdot 5^2 + 1 \cdot 5^1 + 4 \cdot 5^0$$
$$= 2 \cdot \_\_\_ + 1 \cdot \_\_\_ + 4 \cdot \_\_\_$$
$$= \_\_\_ + \_\_\_ + \_\_\_, \text{ or } \_\_\_$$

2. Let's find the base-ten numeral for 156_{eight}. Complete.

$$156_{\text{eight}} = 1 \cdot 8^2 + 5 \cdot 8^1 + 6 \cdot 8^0$$
$$= 1 \cdot \_\_\_ + 5 \cdot \_\_\_ + 6 \cdot \_\_\_$$
$$= \_\_\_ + \_\_\_ + \_\_\_, \text{ or } \_\_\_$$

3. To find the base-five numeral for 148, we divide. Think: the greatest power of 5 in 148 is 5^3, which is 125.

$125\overline{)148}$	$25\overline{)23}$	$5\overline{)23}$	$1\overline{)3}$
$\underline{125}$	$\underline{0}$	$\underline{20}$	$\underline{3}$
23	23	3	0
How many 125's?	How many 25's?	How many 5's?	How many 1's?

So, $148 = 1043_{\text{five}}$.

4. Find base-five numerals.

a. 7 **b.** 24 **c.** 233 **d.** 324

5. To find the base-eight numeral for 156, we divide. Think: the greatest power of 8 in 156 is 8^2, which is 64.

$$\begin{array}{r} 2 \\ 64\overline{)156} \\ \underline{128} \\ 28 \end{array}$$

How many 64's?

$$\begin{array}{r} 3 \\ 8\overline{)28} \\ \underline{24} \\ 4 \end{array}$$

How many 8's?

$$\begin{array}{r} 4 \\ 1\overline{)4} \\ \underline{4} \\ 0 \end{array}$$

How many 1's?

So, $156 = 234_{\text{eight}}$

6. Find base-eight numerals.

a. 10 **b.** 34 **c.** 173 **d.** 200

EXERCISES

Write base-ten numerals.

1. 10_{five} **2.** 30_{five} **3.** 44_{five}

4. 100_{five} **5.** 203_{five} **6.** 444_{five}

7. 7_{eight} **8.** 10_{eight} **9.** 36_{eight}

10. 100_{eight} **11.** 203_{eight} **12.** 427_{eight}

Write base-five numerals.

13. 3 **14.** 5 **15.** 10 **16.** 18 **17.** 24

18. 25 **19.** 37 **20.** 150 **21.** 273 **22.** 200

Write base-eight numerals.

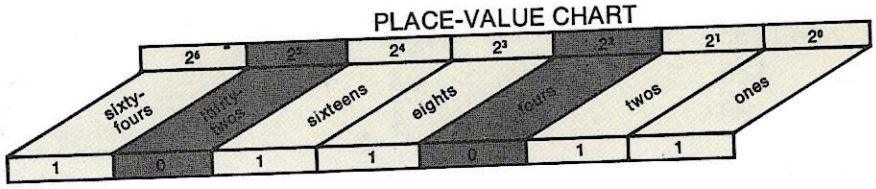
23. 6 **24.** 8 **25.** 16 **26.** 20 **27.** 64

28. 73 **29.** 84 **30.** 100 **31.** 193 **32.** 217

BASE-TWO NUMERATION SYSTEM

The **base-two** or **binary system** has only two digits, 0 and 1. Some computers use this system and pair *off* with 0 and *on* with 1.

PLACE-VALUE CHART



$1011011_{two} = 64 + 0 + 16 + 8 + 0 + 2 + 1$, or 91

1. Study this table. Then continue it through fifteen.

Base Ten	*Base Two*	*Expanded Form Showing The Value of Each Digit*
1	1	1 one
2	10	1 two + 0 ones
3	11	1 two + 1 one
4	100	1 four + 0 twos + 0 ones
5	101	1 four + 0 twos + 1 one

2. What is the base-ten numeral for 10101_{two}? To find out, we use expanded form.

$$\begin{aligned} 10101_{two} &= 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 \\ &= 1 \cdot 16 + 0 \cdot 8 + 1 \cdot 4 + 0 \cdot 2 + 1 \cdot 1 \\ &= 16 + 0 + 4 + 0 + 1, \text{ or } 21 \end{aligned}$$

Find base-ten numerals.

a. 100_{two} **b.** 110_{two} **c.** 11010_{two}

d. 110110_{two} **e.** 110101_{two} **f.** 1000001_{two}

3. To find the base-two numeral for 27, we divide. Think: the greatest power of 2 in 27 is 2^4, which is 16.

$$\begin{array}{r} 1 \\ 16\overline{)27} \\ \underline{16} \\ 11 \end{array} \qquad \begin{array}{r} 1 \\ 8\overline{)11} \\ \underline{8} \\ 3 \end{array} \qquad \begin{array}{r} 0 \\ 4\overline{)3} \\ \underline{0} \\ 3 \end{array} \qquad \begin{array}{r} 1 \\ 2\overline{)3} \\ \underline{2} \\ 1 \end{array} \qquad \begin{array}{r} 1 \\ 1\overline{)1} \\ \underline{1} \\ 0 \end{array}$$

How many 16's? | How many 8's? | How many 4's? | How many 2's? | How many 1's?

So, $27 = 11011_{two}$.

4. Find base-two numerals.

a. 20 **b.** 34 **c.** 37 **d.** 66 **e.** 80

EXERCISES

Write base-ten numerals.

1. 101_{two} **2.** 111_{two} **3.** 1101_{two}

4. 1001_{two} **5.** 10101_{two} **6.** 11111_{two}

7. 101010_{two} **8.** 111000_{two} **9.** 1010110_{two}

10. 11100_{two} **11.** 10110_{two} **12.** 110101_{two}

13. 111111_{two} **14.** 1010100_{two} **15.** 11011011_{two}

Write base-two numerals.

16. 19 **17.** 22 **18.** 25 **19.** 26

20. 30 **21.** 34 **22.** 39 **23.** 48

24. 64 **25.** 70 **26.** 27 **27.** 59

Consider the base-ten and base-two systems.

28. What is the largest possible number that can be named by a four-digit numeral in the base-ten system?

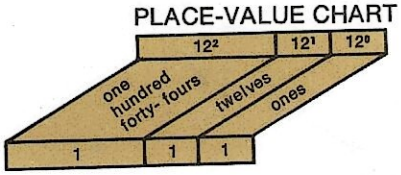
29. What is the largest possible number that can be named by a four-digit numeral in the base-two system?

BASE-TWELVE NUMERATION SYSTEM

We need twelve digits in the **base-twelve system**. Let's call the new digits *T* and *E*.

Base ten	0	1	2	3	4	5	6	7	8	9		
Base twelve	0	1	2	3	4	5	6	7	8	9	*T*	*E*

PLACE-VALUE CHART



1. Study this table. Then continue it through twenty-four.

Base Ten	*Base Twelve*	*Expanded Form Showing the Value of Each Digit*
9	9	9 ones
10	*T*	10 ones
11	*E*	11 ones
12	10	1 twelve + 0 ones
13	11	1 twelve + 1 one

2. What is the base-ten numeral for $T0E_{\text{twelve}}$? To find out we use expanded form.

$$\begin{aligned} T0E_{\text{twelve}} &= T \cdot 12^2 + 0 \cdot 12^1 + E \cdot 12^0 \\ &= 10 \cdot 144 + 0 \cdot 12 + 11 \cdot 1 \\ &= 1{,}440 + 0 + 11, \text{ or } 1{,}451 \end{aligned}$$

Find base-ten numerals.

a. 31_{twelve} **b.** 60_{twelve} **c.** $3E_{\text{twelve}}$

d. $6T_{\text{twelve}}$ **e.** EE_{twelve} **f.** $2TT_{\text{twelve}}$

3. To find the base-twelve numeral for 159, we divide. Think: the greatest power of 12 in 159 is 12^2, which is 144.

$$\begin{array}{r} 1 \\ 144\overline{)159} \\ \underline{144} \\ 15 \end{array} \qquad \begin{array}{r} 1 \\ 12\overline{)15} \\ \underline{12} \\ 3 \end{array} \qquad \begin{array}{r} 3 \\ 1\overline{)3} \\ \underline{3} \\ 0 \end{array}$$

How many 144's? How many 12's? How many 1's?

so, $159 = 113_{\text{twelve}}$.

Find base-twelve numerals.

a. 37 **b.** 30 **c.** 150 **d.** 290

EXERCISES

Write base-ten numerals.

1. 29_{twelve} **2.** 47_{twelve} **3.** 93_{twelve}

4. $2T_{\text{twelve}}$ **5.** $3E_{\text{twelve}}$ **6.** $4T_{\text{twelve}}$

7. 134_{twelve} **8.** $20T_{\text{twelve}}$ **9.** 416_{twelve}

10. 500_{twelve} **11.** $13E_{\text{twelve}}$ **12.** $24T_{\text{twelve}}$

13. 690_{twelve} **14.** $4T3_{\text{twelve}}$ **15.** TET_{twelve}

Write base-twelve numerals.

16. 26 **17.** 31 **18.** 42 **19.** 65

20. 100 **21.** 150 **22.** 289 **23.** 120

Consider the base-ten and base-twelve systems.

24. What is the largest possible number that can be named by a two-digit numeral in the base-ten system?

25. What is the largest possible number that can be named by a two-digit numeral in the base-twelve system?

CHAPTER REVIEW

Finite set or infinite set? [vi]

1. {all people in the world} **2.** $\{6, 7, 8, 9, \ldots\}$

Let A {1, 2, 3, 4, 5}, $B = \{1, 2, 3\}$, $C = \{4, 5\}$, and $D = \{3, 2, 1\}$. True or false? [vi, 2]

3. $2 \in B$ **4.** $B \subseteq A$ **5.** $B = D$ **6.** $5 \notin C$

Let $I = \{1, 4, 7, 9\}$, $J = \{7, 8, 9\}$, and $K = \{1, 2, 10\}$. Find. [4, 6]

7. $I \cap J$ **8.** $J \cup K$ **9.** $I \cap J \cap K$ **10.** $I \cup J \cup K$

11. Which are disjoint, I and J or J and K? [4]

Let $U = \{1, 2, 3, 4, 5, 6, 7\}$ and $A = \{1, 2, 3, 4\}$. Find. [7]

12. $\overline{A}$ **13.** $A \cup \overline{A}$ **14.** $A \cap \overline{A}$ **15.** $n(\overline{A})$

16. Find four subsets of $Z = \{4, 5\}$. [2] [2]

17. Write the standard numeral for $4 \cdot 10^6 + 9 \cdot 10^5 + 9 \cdot 10^4 + 9 \cdot 10^3 + 0 \cdot 10^2 + 0 \cdot 10^1 + 9 \cdot 10^0$. [14]

18. Use exponents to write an expanded numeral for 384,192. [14]

19. Round 4,219,864 to the nearest hundred thousand. [16]

20. Round 4,219,864 to the nearest million. [16]

Write base-ten numerals.

21. 1010_{two} [24] **22.** 116_{eight} [22] **23.** 302_{five} [22] **24.** MCCIV [18]

Write the numeral for 48 in these numeration systems. [22, 24]

25. Base two **26.** Base eight **27.** Base five

CHAPTER TEST

Finite set or infinite set?

1. $\{0, 1, 2, \ldots\}$ **2.** {all students in school}

Let $A = \{1, 2, 3, 4, 5\}$, $B = \{4, 5\}$, $C = \{5, 4\}$, and $D = \{1, 3\}$. True or false?

3. $4 \in B$ **4.** $B = D$ **5.** $C \subseteq A$ **6.** $2 \notin D$

Let $H = \{2, 5, 8, 9, 10\}$, $I = \{6, 8, 10\}$, and $J = \{2, 5, 10\}$. Find.

7. $H \cap I$ **8.** $H \cup I$ **9.** $H \cap I \cap J$

10. Which is true, $I \subseteq H$ or $J \subseteq H$?

11. Are the sets H and I disjoint?

Let $U = \{5, 10, 15, 20, 25, 30, 35\}$ and $K = \{5, 10\}$. Find.

12. $\overline{K}$ **13.** $K \cup \overline{K}$ **14.** $n(\overline{K})$

15. Write the standard numeral for $9 \cdot 10^5 + 8 \cdot 10^4 + 7 \cdot 10^3 + 6 \cdot 10^2 + 5 \cdot 10^1 + 3 \cdot 10^0$

16. Use exponents to write an expanded numeral for 140,346.

17. Round 3,415,000 to the nearest million.

18. Round 24,523,192 to the nearest ten thousand.

Write base-ten numerals.

19. 314_{five} **20.** 101011_{two} **21.** 124_{five} **22.** MDCCIX

Write the numeral for 50 in these numeration systems.

23. Base two **24.** Base five **25.** Base eight

2 THE DECIMAL SYSTEM

MULTIPLYING POWERS OF TEN

Examine this table.

Multiply	*Meaning*	*Product*
$10^2 \cdot 10^1$	$(10 \cdot 10) \cdot 10$	10^3
$10^2 \cdot 10^2$	$(10 \cdot 10) \cdot (10 \cdot 10)$	10^4
$10^2 \cdot 10^3$	$(10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$	10^5
$10^2 \cdot 10^4$	$(10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10)$	10^6
$10^2 \cdot 10^5$	$(10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	10^7
$10^3 \cdot 10^4$	$(10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10)$	10^7

1. Let's multiply: $10^4 \cdot 10^2$.

a. Rename $10^4 \cdot 10^2$ as a product of tens.

b. How many times is 10 used as a factor?

c. Complete: $10^4 \cdot 10^2 = 10^{\square}$.

2. Multiply.

a. $10^3 \cdot 10^3$ **b.** $10^1 \cdot 10^6$ **c.** $10^5 \cdot 10^3$

To multiply powers of ten, add the exponents.

$10^3 \cdot 10^1 = 10^{3+1}$
$= 10^4$

$10^6 \cdot 10^9 = 10^{6+9}$
$= 10^{15}$

3. Make true sentences.

a. $10^6 \cdot 10^{10} = 10^{6+\square}$ **b.** $10^{25} \cdot 10^{\square} = 10^{40}$

4. Multiply.

a. $10^7 \cdot 10^9$ **b.** $10^3 \cdot 10^0$ **c.** $10^a \cdot 10^2$

EXERCISES

Write as products of tens.

1. $10^2 \cdot 10^5$ **2.** $10^5 \cdot 10^2$ **3.** $10^1 \cdot 1$

4. $10^4 \cdot 10^1$ **5.** $10^4 \cdot 10^3$ **6.** $10^3 \cdot 10^4$

Make true sentences.

7. $10^3 \cdot 10^3 = 10^{3+\square}$ **8.** $10^4 \cdot 10^1 = 10^{\square+1}$

9. $10^4 \cdot 10^5 = \square^{4+5}$ **10.** $10^6 \cdot 10 = 10^{\square}$

11. $10^3 \cdot 10^6 = \square^9$ **12.** $\square^3 \cdot \square^2 = 10^5$

13. $10^{12} \cdot 10^{\square} = 10^{25}$ **14.** $10^{\square} \cdot 10^8 = 10^8$

Multiply.

15. $10^2 \cdot 10^2$ **16.** $10^1 \cdot 10^5$ **17.** $10^3 \cdot 10^3$

18. $10^6 \cdot 10^2$ **19.** $10^4 \cdot 10^6$ **20.** $10^7 \cdot 10^6$

21. $10^3 \cdot 10^4$ **22.** $10^5 \cdot 10^7$ **23.** $10^a \cdot 10^3$

24. $10^2 \cdot 10^3$ **25.** $10^3 \cdot 10^2$ **26.** $10^2 \cdot 10^4$

27. $10^4 \cdot 10^2$ **28.** $10^3 \cdot 10^5$ **29.** $10^5 \cdot 10^3$

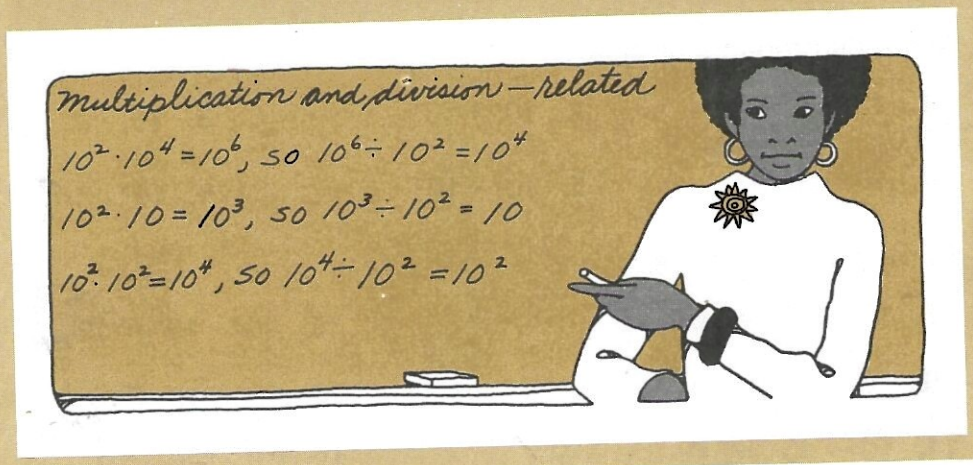
★ **30.** $(10^2 \cdot 10^1) \cdot 10^4$ ★ **31.** $10^2 \cdot (10^3 \cdot 10^4)$

★ **32.** $10^3 \cdot (10^5 \cdot 10^6)$ ★ **33.** $(10^3 \cdot 10^5) \cdot 10^6$

BRAINTEASER

Lee has a collection of records. When he puts them in piles of 2, he has 1 left over. He also has 1 left over when he puts them in piles of 3 or piles of 4. He has none left over when he puts them in piles of 7. What is the least number of records he may have?

DIVIDING POWERS OF TEN


Multiplication and division—related
$10^2 \cdot 10^4 = 10^6$, so $10^6 \div 10^2 = 10^4$
$10^2 \cdot 10 = 10^3$, so $10^3 \div 10^2 = 10$
$10^2 \cdot 10^2 = 10^4$, so $10^4 \div 10^2 = 10^2$

1. Make true sentences.

a. $10^3 \cdot 10^{\square} = 10^5$, so $10^5 \div 10^3 = 10^{\square}$

b. $10^6 \cdot 10^{\square} = 10^8$, so $10^8 \div 10^6 = 10^{\square}$

c. $10^4 \cdot 10^{\square} = 10^4$, so $10^4 \div 10^4 = 10^{\square}$

To divide powers of ten, subtract the exponents.

$10^8 \div 10^3 = 10^{8-3}$
$= 10^5$

$10^{15} \div 10^9 = 10^{15-9}$
$= 10^6$

2. Make true sentences.

a. $10^5 \div 10^4 = 10^{5-\square}$

b. $10^{35} \div 10^{19} = 10^{35-\square}$

3. Divide.

a. $10^5 \div 10^2$ **b.** $10^7 \div 10^4$ **c.** $10^3 \div 10$ **d.** $10^2 \div 10^2$

4. Division may be shown in fractional form.

$6 \div 2 = \frac{6}{2}$
$= 3$

$10^5 \div 10^2 = \frac{10^5}{10^2}$
$= 10^3$

Divide.

a. $\frac{10^5}{10^4}$ **b.** $\frac{10^3}{10^2}$ **c.** $\frac{10^9}{10^4}$ **d.** $\frac{10^4}{10^4}$

EXERCISES

Make true sentences.

1. $10^3 \cdot 10^{\square} = 10^7$, so $10^7 \div 10^3 = 10^{\square}$

2. $10^6 \cdot 10^{\square} = 10^8$, so $10^8 \div 10^6 = 10^{\square}$

3. $10^{\square} \cdot 10^2 = 10^5$, so $10^5 \div 10^2 = 10^{\square}$

4. $10^{\square} \cdot 10^1 = 10^9$, so $10^9 \div 10^1 = 10^{\square}$

5. $\frac{10^6}{10^2} = 10^{6-\square}$

6. $\frac{10^6}{10^4} = \square^{6-4}$

7. $\frac{10^9}{10^3} = \square^6$

8. $\frac{10^7}{10^5} = 10^{\square}$

Make true sentences. Use $+$, $-$, $\times$, or $\div$ for $\equiv$.

9. $10^5 \equiv 10^3 = 10^8$

10. $10^5 \equiv 10^3 = 10^2$

11. $10^6 \cdot 10^4 = 10^{6 \equiv 4}$

12. $10^7 \div 10^3 = 10^{7 \equiv 3}$

Divide.

13. $10^4 \div 10^2$

14. $10^2 \div 10^1$

15. $10^3 \div 10^1$

16. $10^7 \div 10^4$

17. $10^6 \div 10^5$

18. $10^4 \div 10^2$

19. $10^7 \div 10^1$

20. $10^5 \div 10^4$

21. $10^3 \div 10^2$

22. $10^5 \div 10^5$

23. $10^8 \div 10^4$

24. $10^4 \div 10^1$

25. $\frac{10^9}{10^5}$

26. $\frac{10^7}{10^2}$

27. $\frac{10^6}{10^3}$

28. $\frac{10^6}{10^6}$

29. $\frac{10^{15}}{10^8}$

30. $\frac{10^{10}}{10^6}$

31. $\frac{10^{27}}{10^{18}}$

32. $\frac{10^{100}}{10^{10}}$

33. $\frac{10^{27}}{10^9}$

★34. $\frac{10^6 \cdot 10^5}{10^2}$

★35. $\frac{10^7 \cdot 10^3}{10^2}$

★36. $\frac{10^8 \cdot 10^9}{10^5}$

NEGATIVE EXPONENTS

To decide on the meaning of a negative exponent, examine this pattern.

$10^3 = 1{,}000$ Divide by 10.

$10^2 = 100$ Divide by 10.

$10^1 = 10$ Divide by 10.

$10^0 = 1$ Divide by 10.

$10^{\square} = \frac{1}{10}$, or .1

Think: 1 less than 0 is $^{-}1$

$10^{-1} = \frac{1}{10}$, or .1 Divide by 10.

$10^{\square} = \frac{1}{100}$, or .01

Think: 1 less than $^{-}1$ is $^{-}2$

$10^{-2} = \frac{1}{100}$, or .01

1. Give the next three entries in the pattern. Complete.

$10^{\square} = \frac{1}{1{,}000}$, or .001

$10^{\square} = \frac{1}{10{,}000}$, or .0001

$10^{\square} = \frac{1}{100{,}000}$, or .00001

For each natural number n, $10^{-n} = \frac{1}{10^n}$.

$10^{-2} = \frac{1}{10^2}$ $= \frac{1}{100}$, or .01

$10^{-3} = \frac{1}{10^3}$ $= \frac{1}{1{,}000}$, or .001

2. Make true sentences.

a. $10^{-1} = \frac{1}{\square}$ **b.** $10^{\square} = .1$

c. $10^{-5} = \frac{1}{\square}$ **d.** $10^{\square} = .00001$

3. a. What number is equal to 10^3? to 10^{-3}?

b. What number is equal to 10^6? to 10^{-6}?

4. Let's write .01 in exponential form. Think: $.01 = \frac{1}{100}$,

$= \frac{1}{10^2}$, or 10^{-2}

Write in exponential form.

a. 10 **b.** 100 **c.** .1 **d.** .001 **e.** $\frac{1}{10{,}000}$

EXERCISES

Make true sentences.

1. $10^{-1} = \frac{\square}{10}$ **2.** $10^{-2} = \frac{\square}{100}$ **3.** $10^{-3} = \frac{\square}{1{,}000}$

4. $10^{\square} = \frac{1}{10}$ **5.** $10^{\square} = \frac{1}{100}$ **6.** $10^{\square} = \frac{1}{1{,}000}$

7. $10^{\square} = .1$ **8.** $10^{\square} = .01$ **9.** $10^{\square} = .001$

10. $10^{-3} = \square$ **11.** $10^{-5} = \square$ **12.** $10^{-7} = \square$

13. $10^{\square} = \frac{1}{10^8}$ **14.** $10^{\square} = \frac{1}{10^{10}}$ **15.** $10^{\square} = \frac{1}{10^{25}}$

16. $10^2 = 100$; $10^{-2} = \frac{1}{100}$; $10^{-2} = \frac{1}{10^{\square}}$

17. $10^1 = 10$; $10^{-1} = \frac{1}{10}$; $10^{-1} = \frac{1}{10^{\square}}$

★ **18.** $10^2 = 100$; $10^{-2} = \frac{1}{100}$; $10^4 = 10{,}000$; $10^{-4} = \frac{1}{\square}$

★ **19.** $10^3 = 1{,}000$; $10^{-3} = \frac{1}{1{,}000}$; $10^5 = 100{,}000$; $10^{-5} = \frac{1}{\square}$

Write in exponential form.

20. 1 **21.** $\frac{1}{10}$ **22.** $\frac{1}{100}$ **23.** $\frac{1}{1{,}000}$ **24.** $\frac{1}{1{,}000{,}000}$

TUNING UP

Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{10, 20\}$, $C = \{2, 4, 6\}$, and $D = \{20, 10\}$. True or false?

1. $4 \in A$ **2.** $5 \notin B$ **3.** $B \subseteq A$

4. $C \subseteq A$ **5.** $C = B$ **6.** $B = D$

7. $C \neq A$ **8.** $A \subseteq C$ **9.** $8 \in C$

Let $E = \{1, 3, 5, 7, 9\}$, $F = \{5, 6\}$, and $G = \{2, 4, 6\}$. Find.

10. $E \cup F$ **11.** $E \cup G$ **12.** $F \cup G$ **13.** $E \cup F \cup G$

14. $E \cap F$ **15.** $E \cap G$ **16.** $F \cap G$ **17.** $E \cap F \cap G$

Let $U = \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$ and $H = \{10, 50, 100\}$. Find.

18. $\overline{H}$ **19.** $H \cup \overline{H}$ **20.** $H \cap \overline{H}$ **21.** $n(\overline{H})$

Write expanded numerals. Use exponents.

22. 374,684 **23.** 2,794,621 **24.** 750,000,500

Round to the nearest hundred and to the nearest ten thousand.

25. 341,682 **26.** 2,517,084 **27.** 3,684,520

Round to the nearest thousand and to the nearest ten.

28. 341,682 **29.** 2,517,084 **30.** 3,684,520

Write base-ten numerals.

31. 200_{five} **32.** 100011_{two} **33.** 341_{twelve} **34.** MCMLXXIV

Write base-two numerals.

35. 17 **36.** 25 **37.** 35 **38.** 70

Write base-twelve numerals.

39. 26 **40.** 95 **41.** 160 **42.** 250

READING AND WRITING DECIMALS

PLACE VALUES

10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}
tens	ones	tenths	hundredths	thousandths	ten thousandths	hundred thousandths	millionths
3	2	0	0	3	0	5	6

Thirty-two and three thousand fifty-six millionths

1. Read.

a. .9 **b.** .362 **c.** .4568 **d.** 3.70435 **e.** 25.089

2. Write decimals.

a. Twenty-seven thousandths

b. One hundred fifty-four hundred thousandths

c. Two and seven hundredths

d. One hundred and one thousand one ten thousandths

EXERCISES

Write word names.

1. .6 **2.** .600 **3.** .04 **4.** .024

5. 14.123 **6.** 8.0356 **7.** 2.00006 **8.** 1.001234

Write decimals.

9. Seven tenths

10. Eight hundredths

11. Fourteen thousandths

12. Two ten thousandths

13. Nine millionths

14. Two and five tenths

15. Three and three, hundred thousandths

PLACE VALUE

PLACE VALUES

10^6	10^5	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}
millions 1,000,000	hundred thousands 100,000	ten thousands 10,000	thousands 1,000	hundreds 100	tens 10	ones 1	tenths $\frac{1}{10}$, or .1	hundredths $\frac{1}{100}$, or .01	thousandths $\frac{1}{1,000}$ or .001	ten thousandths $\frac{1}{10,000}$, or .0001	hundred thousandths $\frac{1}{100,000}$, or .00001	millionths $\frac{1}{1,000,000}$, or .000001

Standard Numeral

463.78

Expanded Numeral

$4 \cdot 10^2 + 6 \cdot 10^1 + 3 \cdot 10^0 + 7 \cdot 10^{-1} + 8 \cdot 10^{-2}$

1. Complete for the numeral 258.93.

a. The digit 2 has the value $2 \cdot 10^{\square}$.

b. The digit 5 has the value $5 \cdot 10^{\square}$.

c. The digit 8 has the value $8 \cdot 10^{\square}$.

d. The digit 9 has the value $9 \cdot 10^{\square}$.

e. The digit 3 has the value $3 \cdot 10^{\square}$.

2. Write expanded numerals. Use exponents.

a. 2.34 **b.** 381.467 **c.** 4,359.1042

3. Write standard numerals.

a. $2 \cdot 10^2 + 1 \cdot 10^1 + 3 \cdot 10^0 + 4 \cdot 10^{-1}$

b. $2 \cdot 10^0 + 3 \cdot 10^{-1} + 4 \cdot 10^{-2} + 5 \cdot 10^{-3}$

c. $1 \cdot 10^{-1} + 2 \cdot 10^{-2} + 0 \cdot 10^{-3} + 0 \cdot 10^{-4} + 1 \cdot 10^{-5}$

4. The digit 2 in 3.127 has the value $2 \cdot 10^{-2}$, or .02.

Find the values of the underlined digits.

a. 4,561.24 **b.** 3.105 **c.** 24.0263 **d.** 136.045675

EXERCISES

Complete for the numeral 3,648.759.

1. The digit 4 has the value $4 \cdot 10^{\square}$.

2. The digit 7 has the value $7 \cdot 10^{\square}$.

3. The digit 6 has the value $6 \cdot 10^{\square}$.

4. The digit 5 has the value $5 \cdot 10^{\square}$.

5. The digit 3 has the value $3 \cdot 10^{\square}$.

6. The digit 9 has the value $9 \cdot 10^{\square}$.

7. The digit 8 has the value $8 \cdot 10^{\square}$.

Write expanded numerals. Use exponents.

8. 13.4

9. 253.35

10. 4,124.18

11. 6,831.423

12. 27,340.041

13. 37,684.9312

Write standard numerals.

14. $6 \cdot 10^{0} + 3 \cdot 10^{-1} + 0 \cdot 10^{-2} + 3 \cdot 10^{-3}$

15. $0 \cdot 10^{0} + 2 \cdot 10^{-1} + 3 \cdot 10^{-2} + 5 \cdot 10^{-3}$

16. $0 \cdot 10^{0} + 5 \cdot 10^{-1} + 6 \cdot 10^{-2} + 0 \cdot 10^{-3} + 7 \cdot 10^{-4}$

17. $2 \cdot 10^{0} + 0 \cdot 10^{-1} + 0 \cdot 10^{-2} + 0 \cdot 10^{-3} + 5 \cdot 10^{-4} + 6 \cdot 10^{-5} + 7 \cdot 10^{-6}$

Find the values of the underlined digits.

18. 3,126.012

19. 5.346

20. 6.12345

21. 0.432

22. 7.0134

23. 2.00003

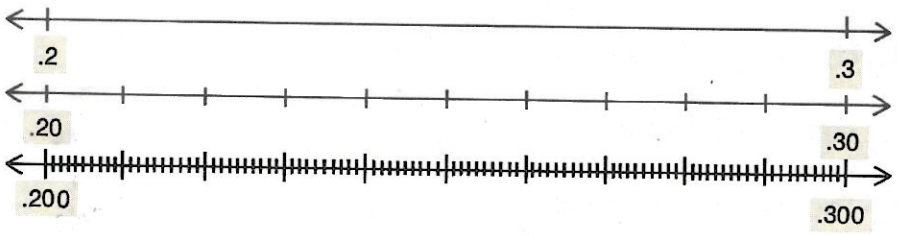
24. 4,563.824

25. 69,350.2136

26. 48,480.2384

COMPARING DECIMALS

We can see that .2 = .20 = .200 and .3 = .30 = .300.



Let's compare .2 and .24.

We know .2 = .20, and .20 < .24. So, .2 < .24.

1. Let's compare .4 and .04.

a. Rename .4 as hundredths.

b. Complete. Use <, >, or =.

.40 ☐ .04, so .4 ☐ .04

2. Compare.

a. .5 and .50 **b.** .26 and .2 **c.** 3.2 and .32

3. Let's compare .1 and .001.

We know .1 = .100, and .100 > .001. So, .1 > .001.

Compare.

a. .6 and .395 **b.** .17 and .170 **c.** 2.1 and 2.001

4. For all numbers *a* and *b*, exactly one of these is true.

$a < b$ $a = b$ $a > b$

Make true sentences. Use =, <, or >.

a. .34 ☐ .340 **b.** .24 ☐ .7 **c.** .4 ☐ .391

d. 4.7 ☐ 4.07 **e.** .26 ☐ .260 **f.** 5.7 ☐ 5.007

5. Let's list .03, .437, and .0024 in order from the largest to the smallest.

We know .03 = .0300 .437 = .4370 .0024 = .0024.
And .4370 > .0300 and .0300 > .0024.
So, .437 > .03 > .0024.

List in order from the largest to the smallest.

a. .007, .0059, .37 **b.** .0058, .58, .058

EXERCISES

Compare.

1. .8 and .08 **2.** .8 and .800 **3.** 3.14 and 3.014

4. .48 and .4 **5.** .9 and .995 **6.** 1.37 and 1.3786

Make true sentences. Use >, <, or =.

7. .24 ☰ .13 **8.** .7 ☰ .6 **9.** .37 ☰ .370

10. .3 ☰ .19 **11.** .4 ☰ .299 **12.** .1 ☰ .09

13. .31 ☰ .4 **14.** .279 ☰ .3 **15.** .4 ☰ .217

16. .2 ☰ .39 **17.** .001 ☰ .100 **18.** .22 ☰ .220

19. .4 ☰ .04 **20.** .31 ☰ .2 **21.** .1 ☰ .999

22. .19 ☰ .190 **23.** .21 ☰ .9163 **24.** 2 ☰ .00002

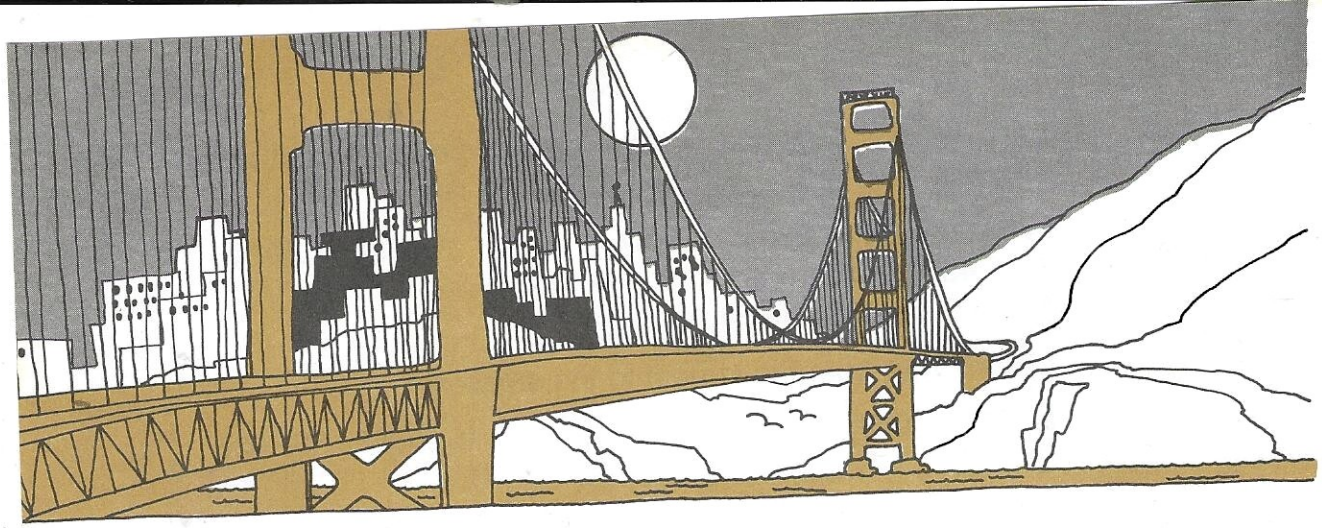
25. 3 ☰ 3.000 **26.** 6 ☰ .793 **27.** 7 ☰ 7.314

List in order from the largest to the smallest.

28. .017, .0343, .17 **29.** .0036, .036, .36

30. .1, .999, .9999 **31.** .91, .091, .0091

32. .02, .0222, .022 **33.** .015, 1.5, .0015



SAN FRANCISCO

1. The Spanish built Yerba Buena in 1834. In 1846, during the Mexican War, the town was captured by the United States Navy. It was renamed San Francisco in 1847. How many years was it from the building of Yerba Buena until it was named San Francisco?

2. San Francisco was almost totally destroyed by earthquake and fire in 1906. It was quickly rebuilt and had a population of 416,912 in 1910. The opening of the Panama Canal helped San Francisco grow to 634,394 by 1930. How much did the population increase in the 20-year period?

3. Cities do not always increase in population. In 1950, the population of San Francisco was 775,357. But, in 1970 the population was 704,209. How much did the population decrease from 1950 to 1970?

4. San Francisco is a beautiful city with water on three sides. It has one of the longest suspension bridges in the world—the Golden Gate Bridge. This bridge has a span of 4,200 feet. How much less than one mile is it?

5. San Francisco's temperatures do not vary much. From January to December the temperatures were 51°, 53°, 55°, 56°, 57°, 59°, 59°, 60°, 62°, 61°, 58°, 53°. Find the average temperature.

ROUNDING DECIMALS

Let's round 4.506 to the nearest tenth.

4.5 0 6

This digit is less than 5.

So, 4.5 0 6 ≐ 4 . 5 .

Let's round 4.506 to the nearest hundredth.

4.5 0 6

This digit is 5 or more.

So, 4.5 0 6 ≐ 4.5 1.

1. Round to the nearest tenth and to the nearest hundredth.

a. 3.156 **b.** 12.249 **c.** 4.38628 **d.** 3.0167

2. 5.28 ≐ 5.3 means that 5.28 was rounded to the nearest tenth. To what place has each been rounded?

a. .736 ≐ .7 **b.** 2.485 ≐ 2.49 **c.** 6.0302 ≐ 6.030

EXERCISES

Round to the nearest tenth and to the nearest hundredth.

1. .78	**2.** .65	**3.** 1.34	**4.** 2.56
5. 3.55	**6.** 4.238	**7.** 7.481	**8.** 8.2568
9. .734	**10.** .375	**11.** 1.241	**12.** 2.348
13. 3.425	**14.** 1.302	**15.** 5.308	**16.** 4.5678

To what place has each been rounded?

17. .24 ≐ .2 **18.** .628 ≐ .63 **19.** 4.30069 ≐ 4.301

20. 6.017 ≐ 6.0 **21.** 8.3006 ≐ 8.301 **22.** 8.3006 ≐ 8.30

SCIENTIFIC NOTATION

Scientists have a special way of naming numbers.

Standard Numeral	*Scientific Notation*
1,000,000	1×10^6
2,000,000	2×10^6
90,000,000,000	9×10^{10}

Number from 1 to 10 ↑ ↑ Power of 10

1. Which are written in scientific notation?

a. 7×10^8 **b.** 23×10^4 **c.** 6×5^3 **d.** 9×10^9

2. Write in scientific notation.

a. 7,000 **b.** 900,000 **c.** 10,000,000 **d.** 8,000,000,000

3. Let's write 270 in scientific notation. Think: 270 is between 200 and 300.

$200 = 2 \times 10^2$
$300 = 3 \times 10^2$ ← $270 = 2.7 \times 10^2$

Write in scientific notation.

a. 740 **b.** 7,400 **c.** 82,000 **d.** 8,200,000

4. Here is a quick way.

$27{,}000 = 2.7 \times 10^{\square}$
$2\,7.0\,0\,0 = 2.7 \times 10^{\square}$

We "moved" four places to the left to get the decimal point in **standard position**. So, $27{,}000 = 2.7 \times 10^4$.

Make true sentences.

a. $63{,}000 = 6.3 \times 10^{\square}$ **b.** $842{,}000{,}000{,}000 = \_\_ \times 10^{11}$

5. Write in scientific notation.

a. 2,400,000 **b.** 374,000,000,000,000

EXERCISES

Which are written in scientific notation?

1. 3×10^8 **2.** 2.5×10^6 **3.** 13×10^6 **4.** 7×3^5

Make true sentences.

5. $4{,}000{,}000 = 4 \times 10^{\square}$ **6.** $8{,}000{,}000 = 8 \times 10^{\square}$

7. $64{,}000 = 6.4 \times 10^{\square}$ **8.** $840{,}000 = \_\_\_\_ \times 10^5$

9. $93{,}000{,}000 = \_\_\_\_ \times 10^7$ **10.** $420{,}000{,}000 = 4.2 \times 10^{\square}$

11. $8{,}430 = 8.43 \times 10^{\square}$ **12.** $869{,}000 = \_\_\_\_ \times 10^5$

13. $3{,}420{,}000 = 3.42 \times 10^{\square}$ **14.** $439{,}000{,}000 = \_\_\_\_ \times 10^8$

Write in scientific notation.

15. 800 **16.** 8,000

17. 80,000 **18.** 800,000

19. 8,000,000 **20.** 800,000,000

21. 3,400 **22.** 34,000

23. 34,000,000 **24.** 340,000,000

25. 3,940 **26.** 39,400,000,000

27. 412,000 **28.** 724,000,000

29. 213,000,000,000 **30.** 614,000,000,000,000,000,000,000

31. The world's largest wind tunnel can move about 60,000,000 cubic feet of air in a minute.

32. The remotest heavenly body visible to the naked eye is about 13,000,000,000,000,000,000 miles from Earth.

STANDARD NUMERALS

The airline distance from New York to Los Angeles is about 2.5×10^3 miles.

Let's write the standard numeral for 2.5×10^3.

We know $10^3 = 1{,}000$. So, $2.5 \times 10^3 = 2{,}500$.

1. Let's write the standard numeral for 8.2×10^5.

Think: 8.2×10^5 is between 8×10^5 and 9×10^5.

$8 \times 10^5 = 800{,}000$

$9 \times 10^5 = 900{,}000$ ← $8.2 \times 10^5 = 820{,}000$

Write standard numerals.

a. 4.2×10^5 **b.** 9.3×10^7 **c.** 8.123×10^8

2. Here is a quick way.

$8.3 \times 10^5 = \square$

↑ Standard position of decimal point

Notice that the exponent is 5, so "move" the decimal point five places to the right.

8.3 0 0 0 0 So, $8.3 \times 10^5 = 830{,}000$.

Write standard numerals.

a. 2.9×10^6 **b.** 9×10^9 **c.** 2.117×10^{10}

EXERCISES

Write standard numerals.

1. 7×10^4 **2.** 6×10^5 **3.** 7.1×10^6

4. 9.3×10^8 **5.** 4.2×10^5 **6.** 8.9×10^7

7. 4.13×10^7 **8.** 8.29×10^6 **9.** 3.41×10^8

10. 4.12×10^8 **11.** 8.231×10^9 **12.** 1.111×10^{10}

4. A metal part is to be .334″ long. The part can be used if it is as much as .005″ shorter or longer. This is shown as .334″ ± .005″. Let's find the lengths that can be used.

Smallest Size	*Largest Size*
.334″	.334″
−.005″	+.005″
.329″	.339″

Any length from .329″ to .339″ can be used.

Find the lengths that can be used.

a. .756″ ± .012″ **b.** .7 m ± .1 m **c.** .34 cm ± .05 cm

EXERCISES

Subtract.

1. .34 − .23	**2.** .549 − .204	**3.** .8437 − .2430	**4.** 12.68 − 4.79
5. .704 − .624	**6.** 23.7 − 5.8	**7.** 4.003 − 2.349	**8.** 38.623 − 29.976
9. 4.23 − 1.317	**10.** 2.4 − 1.235	**11.** 4 − 2.349	**12.** 37.02 − 19.985

13. 4 − 2.3 **14.** 1.57 − .79 **15.** 1.469 − .070

16. 7 − 1.03 **17.** 6.3 − 4.234 **18.** 8 − 4.7

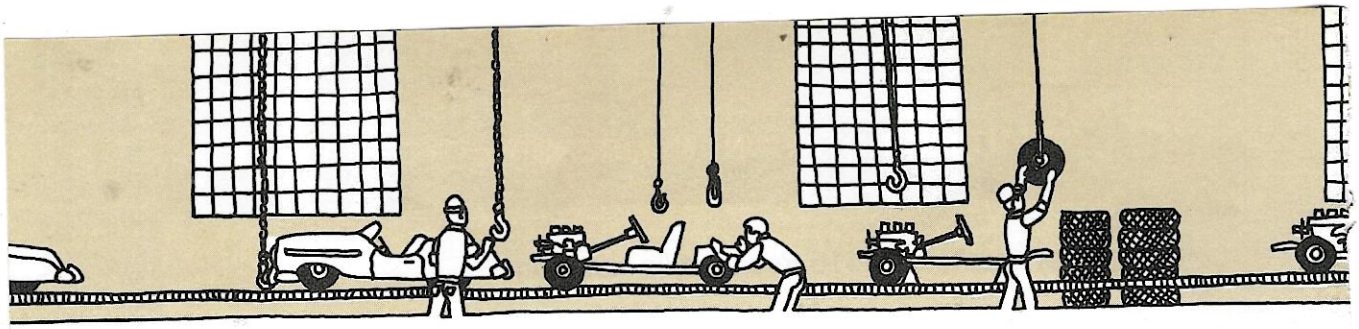
Find the lengths that can be used.

19. .764″ ± .005″ **20.** 2.346 m ± .012 m **21.** .764 km ± .005 km

Solve these problems.

22. A machine part is to be .934″ ± .005″ long. Can a part with a measurement of .930″ be used?

23. A machine part is to be .951 cm ± .013 cm long. Can a part with a measurement of .931″ be used?



COST OF LIVING

WORKING TIME NEEDED TO BUY CERTAIN ITEMS

Item	*Hours*	*Minutes*
Hamburger (1 pound)	0	12
Chicken (1 pound)	0	8
Man's shirt	1	55
Boy's sneakers	1	51
Woman's wool skirt	2	21
Doctor's house visit	3	45

1. Which is cheaper, hamburger or chicken?

2. How many pounds of hamburger can be bought with the money earned in 1 hour?

3. How many minutes must the average production worker work in order to buy a woman's wool skirt?

4. How many minutes must the average production worker work in order to buy a pair of boy's sneakers?

5. How many minutes less than 2 hours must the average production worker work in order to buy a man's shirt?

6. Is a doctor's house visit more or less expensive than the total cost of a pair of boy's sneakers and a man's shirt?

7. In 1919, the average cost of 1 pound of steak was $.39. Fifty years later, the average cost was $1.27. The 1969 cost is about how many times the 1919 cost?

CHAPTER REVIEW

Multiply or divide. [30, 32]

1. $10^2 \cdot 10^4$ **2.** $10^3 \cdot 10^1$ **3.** $10^m \cdot 10^3$

4. $10^7 \div 10^5$ **5.** $10^4 \div 10^1$ **6.** $\frac{10^6}{10^2}$

Make true sentences. [34]

7. $10^0 =$ ____ **8.** $10^{-1} =$ ____ **9.** $10^{-3} = \frac{1}{10^{\square}}$

Write decimals. [37]

10. Twelve and six tenths **11.** One and four millionths

Write expanded numerals. Use exponents. [38]

12. 9.8 **13.** 21.483 **14.** 51,624.2479

Write standard numerals. [38]

15. $4 \cdot 10^0 + 3 \cdot 10^{-1} + 6 \cdot 10^{-2} + 7 \cdot 10^{-3}$

16. $8 \cdot 10^1 + 9 \cdot 10^0 + 6 \cdot 10^{-1} + 8 \cdot 10^{-2} + 5 \cdot 10^{-3}$

Make true sentences. Use $<$, $>$, or $=$. [40]

17. .34 ≣ .09 **18.** .7 ≣ .700 **19.** .642 ≣ .7

Round to the nearest tenth. [43]

20. .38 **21.** 2.52 **22.** 16.4609

Write in scientific notation. [44, 47]

23. 800 **24.** 24,000 **25.** 712,000,000

26. .37 **27.** .0009 **28.** .00000042

Write standard numerals. [46, 47]

29. 8×10^4 **30.** 4.6×10^5 **31.** 3.37×10^8

32. 3.1×10^{-2} **33.** 4.6×10^{-5} **34.** 4.3×10^{-8}

CHAPTER TEST

Multiply or divide.

1. $10^5 \cdot 10^3$ **2.** $10^4 \cdot 10^1$ **3.** $10^a \cdot 10^4$

4. $10^5 \div 10^2$ **5.** $10^6 \div 10^1$ **6.** $\frac{10^5}{10^3}$

Make true sentences.

7. $10^{\square} = 1$ **8.** $10^{\square} = .1$ **9.** $10^{\square} = \frac{1}{10^2}$

Write decimals.

10. One and fifteen thousandths **11.** Two and nine millionths

Write expanded numerals. Use exponents.

12. 12.7 **13.** 63.361 **14.** 41,312.6432

Write standard numerals.

15. $3 \cdot 10^{-1} + 6 \cdot 10^{-2} + 8 \cdot 10^{-3}$

16. $9 \cdot 10^2 + 8 \cdot 10^1 + 6 \cdot 10^0 + 7 \cdot 10^{-1} + 8 \cdot 10^{-2}$

Make true sentences. Use $>$, $<$, or $=$.

17. .6 ≣ .09 **18.** .379 ≣ .4 **19.** .64 ≣ .640

Round to the nearest tenth.

20. .69 **21.** 1.43 **22.** 3.4801

Write in scientific notation.

23. 600 **24.** 3,600 **25.** 240,000,000

26. .84 **27.** .0007 **28.** .000036

Write standard numerals.

29. 6×10^3 **30.** 4.3×10^4 **31.** 8.24×10^6

32. 2.1×10^{-2} **33.** 3.4×10^{-6} **34.** 4.9×10^{-10}

Add.

1. $\begin{array}{r} .7 \\ +.2 \\ \hline \end{array}$ **2.** $\begin{array}{r} .37 \\ +.40 \\ \hline \end{array}$ **3.** $\begin{array}{r} .037 \\ +.421 \\ \hline \end{array}$

4. $\begin{array}{r} 1.4 \\ 3.5 \\ +4.9 \\ \hline \end{array}$ **5.** $\begin{array}{r} 2.36 \\ 4.23 \\ +8.62 \\ \hline \end{array}$ **6.** $\begin{array}{r} 1.421 \\ 9.316 \\ +8.829 \\ \hline \end{array}$

7. $8.3 + .412$ **8.** $.41 + 8.9$ **9.** $81 + 1.36$

Subtract.

10. $\begin{array}{r} 7.3 \\ -2.1 \\ \hline \end{array}$ **11.** $\begin{array}{r} 4.36 \\ -1.04 \\ \hline \end{array}$ **12.** $\begin{array}{r} 8.395 \\ -2.140 \\ \hline \end{array}$

13. $\begin{array}{r} 4.3 \\ -1.6 \\ \hline \end{array}$ **14.** $\begin{array}{r} 7.42 \\ -2.39 \\ \hline \end{array}$ **15.** $\begin{array}{r} 8.004 \\ -2.316 \\ \hline \end{array}$

16. $7 - .06$ **17.** $9.61 - 3.7$ **18.** $4.6 - 2.39$

Multiply.

19. $8 \times .2$ **20.** $8 \times .02$ **21.** $8 \times .002$ **22.** 1.7×3

23. $\begin{array}{r} 2.14 \\ \times 6 \\ \hline \end{array}$ **24.** $\begin{array}{r} 3.041 \\ \times 9 \\ \hline \end{array}$ **25.** $\begin{array}{r} .6 \\ \times .2 \\ \hline \end{array}$ **26.** $\begin{array}{r} .34 \\ \times .7 \\ \hline \end{array}$

27. $\begin{array}{r} 6.741 \\ \times .04 \\ \hline \end{array}$ **28.** $\begin{array}{r} 1.4 \\ \times 2.3 \\ \hline \end{array}$ **29.** $\begin{array}{r} 2.46 \\ \times 3.7 \\ \hline \end{array}$ **30.** $\begin{array}{r} 3.476 \\ \times 1.36 \\ \hline \end{array}$

Divide.

31. $.6 \div 2$ **32.** $.09 \div 3$ **33.** $.120 \div 4$

34. $3\overline{)1.2}$ **35.** $4\overline{)2.08}$ **36.** $6\overline{)3.036}$

37. $7\overline{)2.821}$ **38.** $7\overline{)22.47}$ **39.** $31\overline{).775}$

40. $.6\overline{)12}$ **41.** $.14\overline{)70}$ **42.** $.025\overline{).75}$

43. $.23\overline{)5.75}$ **44.** $3.7\overline{)1.961}$ **45.** $.56\overline{)4.536}$

3 DECIMALS AND MEASUREMENT

ADDING DECIMALS

Adding decimals is like adding whole numbers.

Whole Numbers	*Decimals*
340	3.40
+276	+2.76
616	6.16

Line up decimal points.

Add hundredths to hundredths, tenths to tenths, etc.

1. Add.

a. 3.46 + 2.47

b. 5.731 + .879

c. .346 + .247

d. .2417 + .3162

e. .4 + .6 + .7 + .8

f. 2.3 + 13.7 + 34.6 + 4.8

g. 1.009 + 2.507 + 9.891 + 8.376

h. 12.14 + 23.97 + 38.05 + 29.58

2. Let's add: 15 + 2.1 + 7.43 + 6.764.

15		15.000
2.1		2.100
7.43	or think:	7.430
+ 6.764		+ 6.764
31.294		31.294

Line up decimal points.

Add.

a. 3.4 + 13.72 + 6.004

b. 37.42 + 4.689 + 12.12

c. 3.47 + 2 + 17.513

d. 16 + 7.402 + .0034

EXERCISES

Add.

1.
$$\begin{array}{r} 2.3 \\ 13.7 \\ 34.6 \\ +\ 4.8 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 12.37 \\ 3.61 \\ 14.28 \\ +17.00 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 2.316 \\ 12.413 \\ 3.716 \\ +\ 4.800 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 3.4169 \\ 2.2134 \\ 3.4192 \\ +8.2134 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 11.3 \\ 12.6 \\ 48.0 \\ 29.6 \\ +37.8 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 7.13 \\ 2.47 \\ 36.72 \\ 84.92 \\ +20.00 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 4.702 \\ 46.843 \\ 18.937 \\ 9.247 \\ +83.201 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 7.4136 \\ 8.2004 \\ 9.3090 \\ 6.6748 \\ +5.9216 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 24.3 \\ 1.47 \\ 2.3 \\ +22.463 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 237.423 \\ 24.86 \\ 193.424 \\ +\ 18.7 \\ \hline \end{array}$$

11.
$$\begin{array}{r} 837.42 \\ 84.24 \\ 2.1 \\ +176.837 \\ \hline \end{array}$$

12.
$$\begin{array}{r} 8.003 \\ 16.6476 \\ 12.43 \\ +\ 8.307 \\ \hline \end{array}$$

13.
$$\begin{array}{r} 9.2 \\ 11.03 \\ 23.476 \\ 37.0009 \\ +48.883 \\ \hline \end{array}$$

14.
$$\begin{array}{r} 275.0731 \\ 57.98 \\ 199.99 \\ 38.632 \\ +447.8 \\ \hline \end{array}$$

15.
$$\begin{array}{r} 14.7 \\ 28.53 \\ 316.009 \\ 407.9768 \\ +\ 59.0307 \\ \hline \end{array}$$

16.
$$\begin{array}{r} 526.2 \\ 27.84 \\ 2.37 \\ .005 \\ +\ 1.0007 \\ \hline \end{array}$$

17. $72.3 + 64.8 + 7.9$

18. $.3 + 2.14 + 13.07$

19. $8.1 + 16.3 + 7.402$

20. $12.7 + 25.34 + 49.0096$

21. $15 + 3.47 + .0009$

22. $9.43 + 2 + 1.0503$

23. $8.7 + 2 + 9 + .43$

24. $6.66 + 5.55 + 17$

25. $9.03 + 9.003 + 9.0003$

26. $.01 + .002 + .003 + 4$

27. $11.47 + 5 + 10.004$

28. $13.79 + .538 + 12 + .099$

Solve this problem.

29. A group of scouts hiked 3.7 miles, 4.2 miles, 5.6 miles, and 4.8 miles. How many miles did they hike in all?

SUBTRACTING DECIMALS

Subtracting decimals is like subtracting whole numbers.

Whole Numbers	*Decimals*
516	5.16
−287	−2.87
229	2.29

Line up decimal points.

1. Subtract.

a. 5.1 − 2.8	**b.** .94 − .78	**c.** .094 − .078	**d.** 4.7 − 3.9
e. 23.7 − 5.8	**f.** 2.34 − 1.06	**g.** .680 − .247	**h.** 3.700 − 1.406

2. Let's subtract: 4.5 − 2.34.

4.5	or think:	4.50
−2.34		−2.34
2.16		2.16

Line up decimal points.

Subtract.

a. 4.1 − 2.34	**b.** 7.8 − 2.43	**c.** 8.356 − 4.6
d. 4.35 − 2.126	**e.** 8.34 − 6	**f.** 13.7 − 2.04

3. We can subtract a decimal from a whole number.

Example 5 − 3.67

5.00
−3.67
1.33

Subtract.

a. 8 − 5.069	**b.** 9 − 3.436	**c.** 12 − 11.8
d. 15 − 13.63	**e.** 28 − 14.05	**f.** 36 − 28.804

4. A metal part is to be .334″ long. The part can be used if it is as much as .005″ shorter or longer. This is shown as .334″ ± .005″. Let's find the lengths that can be used.

Smallest Size	*Largest Size*
.334″	.334″
−.005″	+.005″
.329″	.339″

Any length from .329″ to .339″ can be used.

Find the lengths that can be used.

a. .756″ ± .012″ **b.** .7 m ± .1 m **c.** .34 cm ± .05 cm

EXERCISES

Subtract.

1. .34 − .23	**2.** .549 − .204	**3.** .8437 − .2430	**4.** 12.68 − 4.79
5. .704 − .624	**6.** 23.7 − 5.8	**7.** 4.003 − 2.349	**8.** 38.623 − 29.976
9. 4.23 − 1.317	**10.** 2.4 − 1.235	**11.** 4 − 2.349	**12.** 37.02 − 19.985

13. 4 − 2.3 **14.** 1.57 − .79 **15.** 1.469 − .070

16. 7 − 1.03 **17.** 6.3 − 4.234 **18.** 8 − 4.7

Find the lengths that can be used.

19. .764″ ± .005″ **20.** 2.346 m ± .012 m **21.** .764 km ± .005 km

Solve these problems.

22. A machine part is to be .934″ ± .005″ long. Can a part with a measurement of .930″ be used?

23. A machine part is to be .951 cm ± .013 cm long. Can a part with a measurement of .931″ be used?

MULTIPLYING DECIMALS

Examine the pattern.

$314 \times 7 = 2{,}198$
$314 \times .7 = 219.8$
$31.4 \times .7 = 21.98$
$3.14 \times .7 = 2.198$
$.314 \times .7 = .2198$
$.314 \times .07 = .02198$

The number of decimal places in a product is equal to the number of decimal places in the first factor plus the number of decimal places in the second factor.

1. Let's multiply: 3.47×5. We can locate the decimal point in the product by estimating.

Whole numbers	*Decimals*
347	3.47 Estimate: 3.47×5 is
$\times 5$	$\times 5$ between 3×5 and 4×5.
1,735	1735

Insert the decimal point here.

Multiply.

a. 3.213 $\times 4$ **b.** .14 $\times .6$ **c.** .132 $\times .79$ **d.** 42.17 $\times .298$

2. Let's multiply: $1.34 \times .6$.

Whole numbers	*Decimals*
134	1.34 2 decimal places
$\times 6$	$\times .6$ + 1 decimal place
804	804 3 decimal places

Insert the decimal point here.

Multiply.

a. 24.2 $\times 3$ **b.** .34 $\times .17$ **c.** .034 $\times .17$ **d.** 3.213 $\times .24$

3. Copy the products. Then insert the decimal points.

a. $1.4 \times 3 = 42$ **b.** $5.6 \times .3 = 168$

c. $1.9 \times .003 = 57$ **d.** $9 \times .000007 = 63$

EXERCISES

Multiply.

1. 92.7×8	**2.** 34.52×5	**3.** 2.347×4	**4.** 1.416×9
5. $.47 \times .6$	**6.** $24.7 \times .003$	**7.** $.890 \times .10$	**8.** $.46 \times .29$
9. $387 \times .98$	**10.** 26.33×2.06	**11.** $1.203 \times .217$	**12.** 58.73×7.86
13. $1.47 \times .29$	**14.** 37.62×1.3	**15.** $857 \times .78$	**16.** $.6435 \times .64$

Copy the products. Then insert the decimal points.

17. $1.34 \times .6 = 804$

18. $.346 \times .2 = 692$

19. $.143 \times .06 = 858$

20. $.309 \times .27 = 8343$

21. $12.6 \times 8.3 = 10458$

22. $200 \times .03 = 600$

23. $.015 \times .15 = 225$

24. $.24 \times .04 = 96$

25. $3.6 \times .3 = 108$

26. $.408 \times .32 = 13056$

Solve these problems.

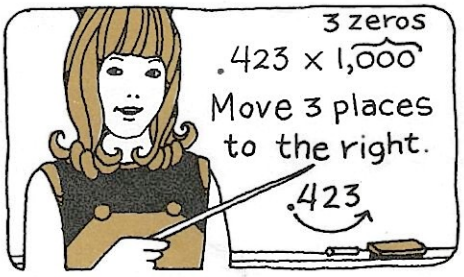
27. It takes Mars 1.88 years to travel around the sun. How long will it take Mars to go around the sun 10 times?

28. A rocket can escape from the surface of Mars if it goes 3.2 miles per second. At this rate, how far will it go in 4.5 seconds?

MULTIPLYING BY POWERS OF 10

Examine the pattern.

$.423 \times 10 = 4.23$
$.423 \times 100 = 42.3$
$.423 \times 1{,}000 = 423$
$.423 \times 10{,}000 = 4{,}230$
$.423 \times 100{,}000 = 42{,}300$



To multiply by powers of 10 such as 10, 100, or 1,000, "move" the decimal point to the right. The number of places to "move" is the same as the number of zeros in the standard numeral for the power of 10.

1. Multiply by "moving" the decimal point.

a. $.083 \times 100$
b. 3.56×10
c. $.257 \times 1{,}000$
d. $1.8793 \times 1{,}000$
e. $5.4612 \times 10{,}000$
f. $.000086 \times 100{,}000$
g. 4.8×10
h. $.0001 \times 1{,}000$
i. 8.62×100
j. $.973 \times 1{,}000$

2. Sometimes we have to insert zeros.

Examples
$.37 \times 1{,}000$ .370 $= 370$
$.37 \times 10{,}000$ .3700 $= 3{,}700$
$.37 \times 100{,}000$ .37000 $= 37{,}000$

Multiply by "moving" the decimal point.

a. $.43 \times 1{,}000$
b. $.3 \times 100$
c. $.083 \times 10{,}000$
d. $.793 \times 100{,}000$
e. $.7 \times 10{,}000$
f. $.007 \times 100{,}000$
g. $3.6 \times 100{,}000$
h. $2.96 \times 10{,}000$
i. $8.62 \times 10{,}000$
j. $.973 \times 100{,}000$

EXERCISES

Multiply by "moving" the decimal point.

1. .08 × 10

2. .08 × 100

3. .08 × 1,000

4. .08 × 10,000

5. .08 × 100,000

6. .296 × 1,000

7. 1.8 × 1,000

8. 3.265 × 10,000

9. 48.6 × 10

10. 68.73 × 100

11. 29.003 × 1,000

12. 87.4 × 1,000

13. .00005 × 10,000

14. .00006 × 100,000

15. 4.63 × 1,000

16. .479 × 10,000

17. 1.98 × 100

18. 37.53 × 1,000

19. .561 × 1,000

20. .658 × 100

21. .972 × 10,000

22. .402 × 100,000

23. .0094 × 1,000

24. .0036 × 10,000

25. .0061 × 100,000

26. .0007 × 100

27. .0009 × 100,000

28. .000001 × 1,000,000

BRAINTEASERS

1. To what number would you add 10 so that the sum is 3 times the number?

2. There is a set of numbers called the **Fibonacci numbers**. The first nine numbers are 1, 1, 2, 3, 5, 8, 13, 21, 34. What are the next five numbers?

DIVIDING DECIMALS

Whole Numbers	*Decimals*		
$4\overline{)8}$	2 tenths $4\overline{)8 \text{ tenths}}$	or	$4\overline{).8}$
$9\overline{)36}$	4 hundredths $9\overline{)36 \text{ hundredths}}$	or	$9\overline{).36}$

If the divisor is a whole number, line up the decimal points in the dividend and the quotient. Then divide as with whole numbers.

1. Divide.

a. $3\overline{).9}$ **b.** $6\overline{).012}$ **c.** $32\overline{)10.24}$

2. Observe this pattern.

$$\begin{aligned} 8 \div 4 &= 2 \\ (8 \cdot 10) \div (4 \cdot 10) &\longrightarrow 80 \div 40 = 2 \\ (8 \cdot 100) \div (4 \cdot 100) &\longrightarrow 800 \div 400 = 2 \end{aligned}$$

What do you notice?

If the dividend and the divisor are multiplied by the same nonzero number, the quotient remains the same.

3. Let's divide: $.45\overline{)1.125}$

a. It's easy to divide when the divisor is a whole number. Multiply .45 by 10. Is the divisor a whole number?

b. Complete: $.45 \times \underline{\quad} = 45.$

c. Multiply 1.125 by the same number. Complete:

$$1.125 \times 100 = \underline{\quad}$$

d. Now divide: $45\overline{)112.5}$

e. Multiply: $2.5 \times .45$. Did you get 1.125?

4. We can use a caret (∧) to "move" the decimal point.

$.27\overline{)915.3}$ $.27\overset{\wedge}{)}915.30\overset{\wedge}{}$ $27\overline{)91{,}530}$

Complete the division.

5. Divide.

a. $1.6\overline{)9.6}$ **b.** $.024\overline{)1.44}$ **c.** $.231\overline{).3696}$

6. Copy the quotients. Then insert the decimal points.

a. $3.7\overline{)1.924}$ 52 **b.** $.56\overline{)4.592}$ 82 **c.** $6.9\overline{).5106}$ 74

EXERCISES

Divide.

1. $7\overline{)2.149}$ **2.** $9\overline{)364.5}$ **3.** $8\overline{)22.48}$

4. $31\overline{)74.4}$ **5.** $82\overline{)1.804}$ **6.** $34\overline{).1020}$

7. $.05\overline{)25}$ **8.** $.12\overline{)3.6}$ **9.** $.6\overline{).012}$

10. $.024\overline{).096}$ **11.** $.024\overline{)96}$ **12.** $.024\overline{).96}$

13. $2.35\overline{)3.995}$ **14.** $45.9\overline{).1377}$ **15.** $.987\overline{)36.519}$

16. $.15\overline{).225}$ **17.** $.027\overline{)8.1}$ **18.** $.351\overline{)7.2306}$

Copy the quotients. Then insert the decimal points.

19. $33\overline{)19.866}$ 602 **20.** $4.7\overline{)13.63}$ 29 **21.** $22.7\overline{).8626}$ 38

22. $.012\overline{).24}$ 2 **23.** $.36\overline{)108}$ 3 **24.** $.0048\overline{)1.44}$ 3

Solve this problem.

25. Susan saves $1.65 a week. How many weeks will it take her to save $64.35?

ROUNDING QUOTIENTS

Scientists and others often have to round quotients.
Let's find the quotient $2.1\overline{)9.08}$ to the nearest tenth.

$2.1\overline{)9.08}$ $\quad$ $2.1\overline{)9.08}$ $\quad$ $21\overline{)90.80}$ $\quad$ $21\overline{)90.80}$ $\quad 4.32 \doteq 4.3$

1. Find each quotient to the nearest tenth.

a. $.6\overline{)4.3}$ $\quad$ **b.** $5\overline{).831}$ $\quad$ **c.** $.56\overline{)3.428}$

2. To find a quotient to the nearest hundredth, we carry the division to thousandths.

$.06\overline{)3418}$ $\quad$ $.06\overline{)34.18}$ $\quad$ $6\overline{)34.180}$ $\quad$ $6\overline{)34.180}$ $\quad 5.696 \doteq 5.70$

Find each quotient to the nearest hundredth.

a. $.7\overline{)3.49}$ $\quad$ **b.** $9\overline{).834}$ $\quad$ **c.** $.47\overline{)5.198}$

EXERCISES

Find each quotient to the nearest tenth.

1. $.4\overline{)3.7}$ $\quad$ **2.** $7\overline{)1.3}$ $\quad$ **3.** $.9\overline{).4}$

4. $2.1\overline{).7}$ $\quad$ **5.** $3.2\overline{)1.5}$ $\quad$ **6.** $.03\overline{).352}$

7. $.47\overline{)2.987}$ $\quad$ **8.** $6.3\overline{)38.6}$ $\quad$ **9.** $1.3\overline{).8}$

Find each quotient to the nearest hundredth.

10. $4\overline{)3.59}$ $\quad$ **11.** $.03\overline{)4.132}$ $\quad$ **12.** $.7\overline{)359.12}$

13. $21\overline{)5.67}$ $\quad$ **14.** $3.1\overline{).6213}$ $\quad$ **15.** $.62\overline{).06224}$

16. $.29\overline{).1286}$ $\quad$ **17.** $5.8\overline{).0379}$ $\quad$ **18.** $2.4\overline{).83}$

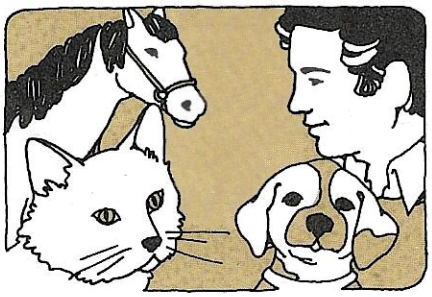
EQUIVALENT AGES OF ANIMALS AND HUMANS

1. We often say that a 4 year old dog is equivalent in age to a 28 year old man. The oldest dog on record lived 34 years. What would be the equivalent age of a human? Do you think this is possible? We see that this method does not always work. However, it does work when a dog is about 4 years old.

Use this table for Problems 2–5.

EQUIVALENT AGES IN YEARS

Man	*Dog*	*Cat*	*Horse*
20	1.2	0.7	4.6
25	2.8	2.6	6.6
30	4.5	4.6	8.7
35	6.2	6.5	10.6
40	7.8	8.4	12.7
45	9.5	10.3	14.8
50	11.2	12.2	16.8



2. Consider a 20 year old person an adult. What are the equivalent ages of a dog, a cat, and a horse which would be considered adults?

3. One dog is equivalent in age to a man of 20. Another dog is equivalent in age to a man of 50. What is the difference in the ages of the dogs?

4. One dog is equivalent in age to a man of 25. Another dog is equivalent in age to a man of 35. What is the difference in the ages of the dogs?

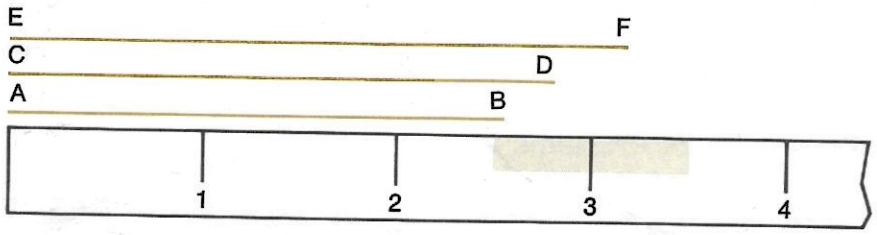
5. A dog is equivalent in age to a man of 20. A cat is equivalent in age to a man of 20. What is the difference in the ages of the dog and the cat?

6. A dog is equivalent in age to a 24 year old man. Use the formula below to find the equivalent age of the dog.

$$\text{Dog's age} = .33 \times \text{man's age} - 5.5$$

MEASUREMENT AND INTERVALS

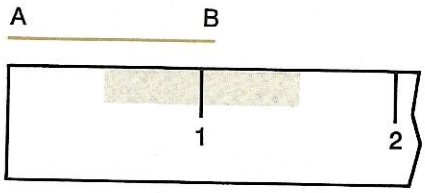
The length of each line segment below falls within the **interval** from $2\frac{1}{2}$ inches to $3\frac{1}{2}$ inches.



The length of each line segment is 3 inches to the nearest inch.

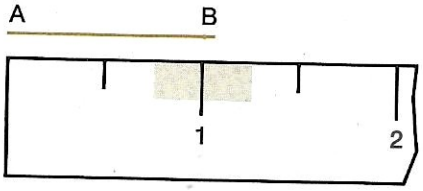
1. a. Within what interval does the length of $\overline{AB}$ fall?

b. What is the length of $\overline{AB}$ to the nearest inch?



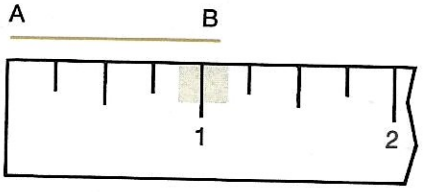
2. a. Within what interval does the length of $\overline{AB}$ fall?

b. What is the length of $\overline{AB}$ to the nearest $\frac{1}{2}$ inch?



3. a. Within what interval does the length of $\overline{AB}$ fall?

b. What is the length of $\overline{AB}$ to the nearest $\frac{1}{4}$ inch?



4. An actual length which is measured as 6 feet to the nearest foot must fall within the interval from $5\frac{1}{2}$ feet to $6\frac{1}{2}$ feet. Within what intervals must the actual lengths fall?

a. $9\frac{1}{4}$ m to the nearest $\frac{1}{4}$ m **b.** 8 cm to the nearest cm

5. All measurements are approximate. The smaller the unit of measurement used, the more **precise** (closer to the actual length) is the measurement. Which measurement is more precise?

a. 3″ or .03″ **b.** 2 mi or 2 ft **c.** $3\frac{1}{2}$ m or $3\frac{2}{4}$m

EXERCISES

Within what intervals must the actual lengths fall?

1. 7 in. to the nearest in.

2. $8\frac{1}{2}$ in. to the nearest $\frac{1}{2}$ in.

3. $9\frac{3}{4}$ m to the nearest $\frac{1}{4}$ m

4. $10\frac{5}{8}$ cm to the nearest $\frac{1}{8}$ cm

Which measurement is more precise?

5. $3\frac{1}{4}''$ or $1\frac{1}{2}''$

6. $5\frac{1}{2}''$ or $12\frac{3}{8}''$

7. $5\frac{1}{8}''$ or $10\frac{1}{10}''$

8. $11\frac{1}{8}''$ or $7\frac{3}{4}''$

9. .07″ or .9″

10. .006″ or .97″

11. .346″ or .1″

12. .52″ or .311″

13. .24″ or .240″

14. .6 sec or .6 min

15. .05 m or .05 mm

16. .001 kg or .001 g

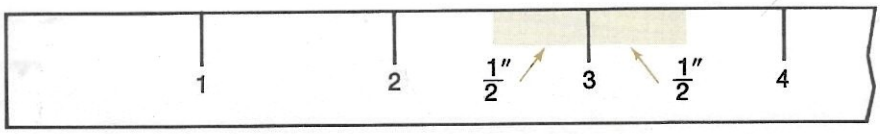
Who used a more precise measurement?

17. Ann measured the length of her banjo to the nearest one-fourth inch. Ann's father measured a piece of wood for a cabinet to the nearest one-eighth inch.

18. Ted measured the thickness of a sheet of paper to the nearest .001 inch. Pam measured the thickness to the nearest .01 inch.

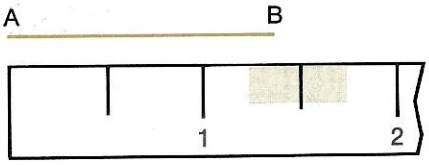
GREATEST POSSIBLE ERROR

The shaded area shows the interval within which a length may be reported as 3 inches to the nearest inch.



We see that the greatest possible difference between the actual length and the reported length is $\frac{1}{2}$ inch. This is called the **greatest possible error**.

1. The measurement of $\overline{AB}$ is $1\frac{1}{2}$ inches to the nearest $\frac{1}{2}$ inch.



a. What is the unit of measurement?

b. What is the smallest possible length $\overline{AB}$ might be?

c. What is the largest possible length $\overline{AB}$ might be?

d. What is the greatest possible error?

The greatest possible error of a measurement is equal to one half of the unit of measurement.

2. Find the greatest possible errors.

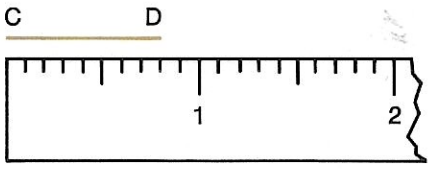
a. 9 inches

b. 3 meters

c. $5\frac{1}{2}$ inches

d. $4\frac{3}{4}$ feet

3. The measurement of $\overline{CD}$ is .8″ to the nearest .1″. Let's find the greatest possible error.



a. What is the unit of measurement?

b. What is $\frac{1}{2}$ of .1″? Think: $2\overline{).1}$. So, the greatest possible error is .05″.

4. Find the greatest possible errors.

a. .3″ **b.** .46″ **c.** .347″ **d.** 5.03″

EXERCISES

Find the greatest possible errors.

1. 17 yd	**2.** 4 ft	**3.** 8 in.
4. 5 m	**5.** 2 cm	**6.** 8 mm
7. $6\frac{3}{4}$ ft	**8.** $18\frac{1}{8}$ yd	**9.** $25\frac{2}{3}$ mi
10. $\frac{5}{16}$ in.	**11.** $13\frac{1}{2}$ cm	**12.** $20\frac{1}{4}$ m
13. .4 in.	**14.** 1.3 in.	**15.** 2.8 in.
16. .003 cm	**17.** .03 mm	**18.** .90 m
19. .430 km	**20.** 6.05 cm	**21.** 8.6 m

TUNING UP

Compute.

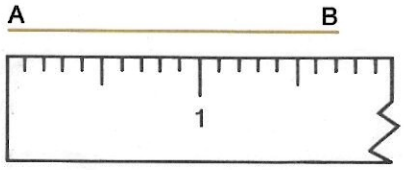
1. $10^7 \cdot 10^2$	**2.** $10^6 \cdot 10$	**3.** $10^8 \cdot 10^9$
4. $10^7 \div 10^2$	**5.** $10^6 \div 10^2$	**6.** $10^9 \div 10^8$

Make true sentences. Use $>$, $<$, or $=$.

7. .4 ≣ .400	**8.** .39 ≣ .4	**9.** 1 ≣ .9
10. .586 ≣ .6	**11.** .2 ≣ .002	**12.** .01 ≣ .1

FINDING INTERVALS OF MEASUREMENTS

The measurement of $\overline{AB}$ is 1.7″ to the nearest .1″.
Let's find the interval of measurement.



Unit of measurement: .1″
Greatest possible error: .05″

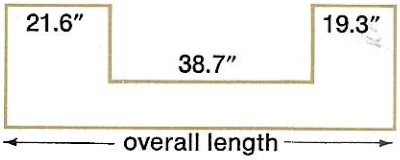
Smallest Possible Length	*Measurement*	*Largest Possible Length*
1.70″		1.70″
− .05″		+ .05″
1.65″	1.7″	1.75″

The length of $\overline{AB}$ is certain to fall within the interval from 1.65″ to 1.75″.

1. Let's find the interval within which the measurement 3.4″ falls.
   - **a.** What is the unit of measurement?
   - **b.** What is the greatest possible error?
   - **c.** Subtract: 3.4 − .05. What is the smallest possible length?
   - **d.** Add: 3.4 + .05. What is the largest possible length?
   - **e.** Within what interval does 3.4″ fall?

2. Within what intervals do these measurements fall?
   a. 2.8″ **b.** 4.35″ **c.** 6.30″

3. Look at the figure on the right. Let's find the interval within which the overall length falls.



a. The unit of measurement for each length is .1″. What is the greatest possible error of each measurement?

b. Complete.

Smallest Possible Length	*Measurement*	*Largest Possible Length*
21.55″	21.6″	21.65″
?	38.7″	?
+ ?	+ 19.3″	+ ?

4. Within what intervals do the sums of the measurements fall?

a.
13.7″
24.6″
+ 18.9″

b.
12.8″
34.6″
+ 81.9″

c.
2.46″
3.47″
+ 2.09″

EXERCISES

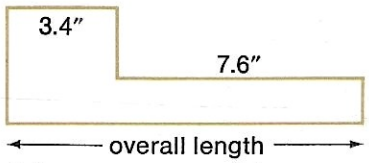
Within what intervals do the actual lengths fall?

1. 4.2″ **2.** 3.9″ **3.** 2.34″ **4.** 4.07″

5. 8.42″ **6.** 5.16″ **7.** 2.134″ **8.** 7.248″

Within what intervals do the overall lengths fall?

9.



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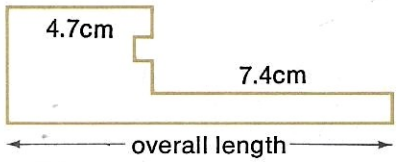
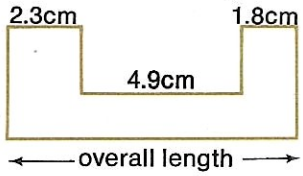
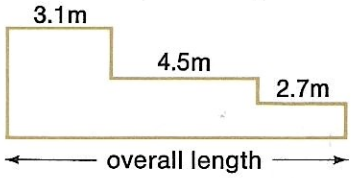


11.



12.



Within what intervals do the sums of the measurements fall?

13.
34.6″
84.7″
+ 12.9″

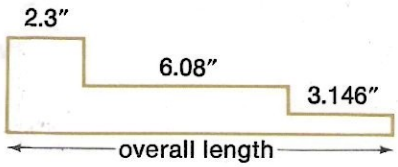
14.
21.3 m
18.4 m
+ 17.6 m

15.
2.53″
7.64″
+ 8.72″

16.
24.12 cm
9.34 cm
+ 8.16 cm

ADDING AND SUBTRACTING MEASUREMENTS

If different units of measurement are used, we need an agreement on how to add the measurements.



The least precise unit of measurement is .1″.

$$\begin{array}{rl} 2.3\ '' & = 2.3'' \\ 6.08\ '' & \doteq 6.1'' \\ +3.146'' & \doteq \underline{3.1''} \\ & 11.5'' \end{array}$$

To add measurements, round each to the least precise unit. Then add. The sum cannot be more precise than the least precise unit.

1. Let's add: $3.4' + 6.89' + 4.89'$.

a. What is the least precise unit of measurement?

b. To what unit should each measurement be rounded?

c. Round each measurement.

d. What is the sum of the measurements?

2. Add the measurements.

a.
$$\begin{array}{r} 5.12 \\ 7.6 \\ +8.7 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 5.36 \\ 7.247 \\ +9.37 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 2.3784 \\ 1.62 \\ +8.416 \\ \hline \end{array}$$

3. Let's subtract: $7.64'' - 2.3''$.

Least precise unit of measurement: .1″
Round each measurement to the nearest .1″.

$$\begin{array}{rl} 7.64'' & \doteq 7.6'' \\ -2.3\ '' & = \underline{2.3''} \\ & 5.3'' \end{array}$$

Subtract the measurements.

a.
$$\begin{array}{r} 3.6 \\ -2.15 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 4.932 \\ -2.78 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 7 \\ -2.43 \\ \hline \end{array}$$

EXERCISES

Add the measurements.

1. 4.26 7.31 + 8.047	**2.** 14.6 8.7 + 9.04	**3.** 13.43 12.7 + 8.94	**4.** 1.9 4.03 + 3.147
5. 34.167 22.34 18.7 + 9.45	**6.** 652.7 24.81 37.846 + 17.70	**7.** 14.003 12.01 23.0 + 7.642	**8.** 8 16.3 7.402 + .0034
9. 24.3 1.47 2.3 + 22.463	**10.** 237.423 24.86 193.424 + 18.7	**11.** 837.42 84.24 2.1 + 176.837	**12.** 3.47 2.1 17.513 + 19.24

Subtract the measurements.

13. 4.71 − 2.6	**14.** 13.1 − 8.094	**15.** 14.7 − 2.81	**16.** 4.38 − 2.6
17. 5.249 − 2.76	**18.** 84.0001 − 2.4	**19.** 8.3 − 4.47	**20.** 5.7 − 2.363
21. 12 − 9.005	**22.** 8.356 − 4.6	**23.** 28.347 − 14.9	**24.** 234.27 − 129.653

Solve these problems.

25. Tom cut a board into 3 pieces. The lengths were 2.7 cm, 8.24 cm, and 9.637 cm. What was the overall length of the board?

26. Jane's history book weighed 3.25 pounds. Her science book weighed 2.625 pounds. What was the difference in their weights?

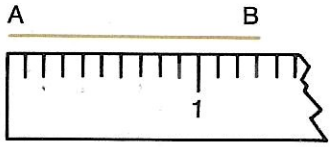
27. Mr. Lo's flagpole is 4.8 meters high. Mr. Hi's flagpole is 5.06 meters high. How much higher is Mr. Hi's flagpole?

SIGNIFICANT DIGITS

Measurement	*Unit*	*Number of Times Unit Is Contained*	*Number of Significant Digits*
200 mi	100 mi	2	1
200 ft	10 ft	20	2
140 yd	1 yd	140	3
9.0 in.	.1 in.	90	2
.06 in.	.01 in.	6	1
.003 in.	.001 in.	3	1

The **significant digits** in a measurement are those digits that indicate the number of times the unit is contained in the measurement.

1. The length of $\overline{AB}$ is 1.3 inches.



a. What unit is used?

b. Divide: $.1\overline{)1.3}$.

c. How many significant digits are there in 1.3 inches?

2. Find the number of significant digits.

a. 2.7 in. **b.** 9 ft **c.** 384 cm **d.** 2,136 m

e. .04 cm **f.** 2.03 m **g.** .035 in. **h.** 5.000 in.

Every nonzero digit is always significant. Zero is sometimes significant and sometimes not significant.

3. Complete.

	Measurement	*Unit*	*Number of Times Unit Is Contained*	*Number of Significant Digits*
a.	6,000 ft	1,000 ft		
b.	6,000 ft	1 ft		
c.	6,000 ft	10 ft		
d.	6,000 ft	100 ft		

EXERCISES

Complete.

	Measurement	*Unit*	*Number of Times Is Contained*	*Number of Significant Digits*
1.	12 mi	1 mi	12	2
2.	7 m	1 m		
3.	103 ft		103	
4.	1,302 yd	1 yd		
5.	205 lb			3
6.	24 cm	1 cm		
7.	.3 in.	.1 in.		
8.	.03 in.	.01 in.		
9.	.003 in.	.001 in.		
10.	.4 kg		4	
11.	.40 mm	.01 mm		
12.	.400 in.		400	
13.	2.3 cm		23	
14.	17.03 cm			4
15.	12.421 in.	.001 in.		
16.	12.4 gal		124	
17.	7.64 mi	.01 mi		

Solve these problems.

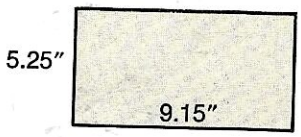
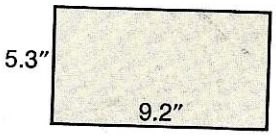
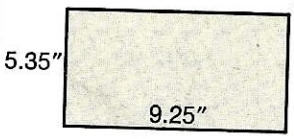
18. The diameter of the earth is 7,920 miles to the nearest 10 miles. How many significant digits are there in this measurement?

19. The distance to the sun is 93,000,000 miles to the nearest 1,000,000 miles. How many significant digits are there in this measurement?

MULTIPLYING MEASUREMENTS

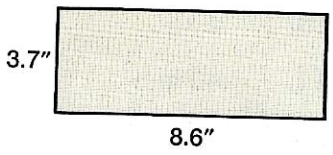
A rectangle has measurements of 5.3″ and 9.2″. Let's find the interval within which its area will fall.

The greatest possible error of each measurement is .05″.

Smallest Possible Length and Width	Measured Length and Width	Largest Possible Length and Width
		

The area falls within the interval from 48.0375 to 49.4875. A good approximation of the area is 49 square inches.

1. Consider this rectangle.



a. What is the greatest possible error in the width? in the length?

b. Within what interval does the width fall? the length?

c. Find the smallest possible area.

d. Find the largest possible area.

e. Within what interval does the area fall?

f. Multiply: 3.7×8.6.

g. Does 31.82 fall within the interval from 31.2075 to 32.4375?

h. How many significant digits are there in 3.7″? in 8.6″? in 31.82 square inches?

i. We shall agree to round the product to the same number of significant digits. Round 31.82 to 2 significant digits.

2. Round to 2 significant digits.

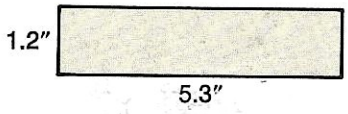
Examples 3.47″ 3.5″
2.123 m 2.1 m

a. 2.43″ **b.** 453 mi **c.** 6.034′ **d.** 7.3471 m

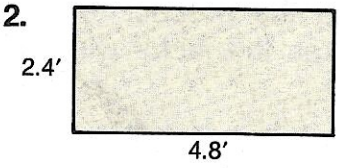
EXERCISES

Find the intervals within which the areas will fall.

1.



2.

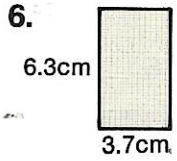


Round to 2 significant digits.

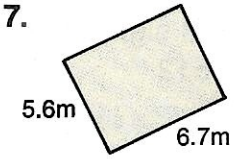
3. 9,357 mi **4.** 2.063″ **5.** .123 cm

Find the areas. Round to 2 significant digits.

6.



7.



Round to 3 significant digits.

8. 4,562 mi **9.** 3.0126″ **10.** 241.34 cm

BRAINTEASERS

1. Give five sets of three numbers each, such that their sum is three times their average.

2. A train one mile long is just entering a tunnel which is one mile long. The train is traveling at the rate of 60 miles an hour. How long will it take the train to go through the tunnel?

A RULE FOR MULTIPLYING MEASUREMENTS

All measurements are approximate. Here is a method that will give a good approximation when multiplying measurements.

1. If necessary, round the factor that has more significant digits so that it has only one more significant digit than the other factor.
2. Multiply.
3. Round the product to the same number of significant digits as the factor with the fewer number of significant digits.

$3.1 \times 2.4 = 7.44$ So, $3.1 \times 2.4 \doteq 7.4$

not necessary to round

$3.12 \times 2.4 = 7.488$ So, $3.12 \times 2.4 \doteq 7.5$.

not necessary to round

3.125×2.4

Round to 3 significant digits.

$3.13 \times 2.4 \doteq 7.512$ So, $3.125 \times 2.4 \doteq 7.5$.

1. Which factors, if any, would you round before multiplying?

a. $2.17'' \times 3.4''$ **b.** $14.38 \text{ m} \times 1.6 \text{ m}$

c. $15 \text{ cm} \times 4.8 \text{ cm}$ **d.** $.0007' \times 2.13'$

2. Round each product to the correct number of significant digits.

a. $3.46 \times .21 = .7266$ **b.** $.35 \times .4 = .140$

c. 71.42×2.2
$71.4 \times 2.2 = 157.08$

d. 53.04×1.2
$53.0 \times 1\,2 = 63.60$

3. Multiply the measurements.

a. 23×1.4 **b.** 53.041×1.23

EXERCISES

Which factors would you round before multiplying?

1. 3.146 m × 7.6 m

2. 4.12 cm × 2.3 cm

3. .123 in. × .156 in.

4. 25 mm × .37 mm

5. 21.37 km × 1.9 km

6. 83.441 m × 2.6 m

Multiply the measurements.

7. 11.4 × 2.3

8. 4.07 × 1.6

9. 2.71 × 1.24

10. 3.016 × 2.1

11. 40.73 × 2.8

12. 173.45 × 4.6

13. 50.001 × .34

14. .712 × .14

TUNING UP

Add.

1. 342.7 + 83.5 + 682.4 + 921.3

2. 356.42 + 89.78 + 64.23 + 193.47

3. 8,413 + 312.08 + 64.1312 + 73

Subtract.

4. 349.76 − 312.06

5. 349.2 − 6.18

6. 8 − 2.49

Multiply.

7. 24.01 × 6

8. 35.29 × 1.2

9. 5.371 × .36

Divide.

10. $3\overline{)36.21}$

11. $.8\overline{)43.44}$

12. $.24\overline{)720.48}$

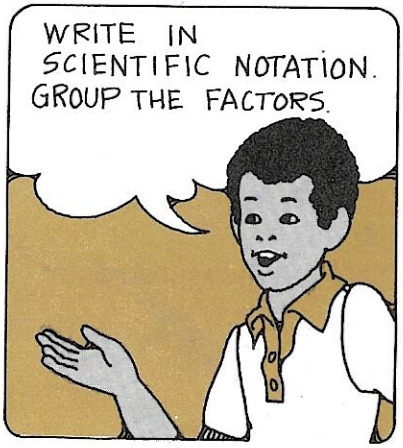
13. Round 34,624,819 to the nearest million.

14. Round 27.24819 to the nearest hundredth.

USING SCIENTIFIC NOTATION

$$
\begin{aligned}
&12{,}000{,}000 \times 700{,}000{,}000 \\
&= 1.2 \times 10^7 \times 7 \times 10^8 \\
&= (1.2 \times 7) \times (10^7 \times 10^8) \\
&= 8.4 \times 10^{15}
\end{aligned}
$$

$$
\begin{aligned}
&\frac{840{,}000{,}000}{21{,}000} \\
&= \frac{8.4 \times 10^8}{2.1 \times 10^4} \\
&= \frac{8.4}{2.1} \times \frac{10^8}{10^4} \\
&= \frac{8.4}{2.1} \times \frac{10^8}{10^4} \\
&= 4 \times 10^4
\end{aligned}
$$



1. Let's multiply: 30,000,000,000 × 20,000.

   a. Write in scientific notation.

   b. Group the factors.

   c. Complete: $(3 \times 2) \times (10^{10} \times 10^4) =$ ____ .

   d. Write the standard numeral for 6×10^{14}.

2. Multiply. Give the answers in scientific notation.

   a. 400,000 × 20,000,000 **b.** 5,000,000,000 × 1,000,000

   c. $4.32 \times 10^6 \times 1.2 \times 10^8$ **d.** $4.32 \times 2.1 \times 10^5$

3. Sometimes we have to rewrite the answer.

$$
\begin{aligned}
&600{,}000 \times 3{,}000{,}000 \\
&= 6 \times 10^5 \times 3 \times 10^6 \\
&= (6 \times 3) \times (10^5 \times 10^6) \\
&= 18 \times 10^{11} \quad \text{Think: 18 is not a number from 1 to 10.} \\
&= 1.8 \times 10 \times 10^{11} \quad \text{Write 18 in scientific notation.} \\
&= 1.8 \times 10^{12}
\end{aligned}
$$

   Multiply. Give the answers in scientific notation.

   a. 400,000 × 4,000,000 **b.** 34,000 × 500,000,000

4. Let's divide: $\frac{8,000,000,000}{200,000}$.

   a. Write in scientific notation.

   b. Group the factors.

   c. Simplify.

   d. Write the standard numeral for 4×10^4.

EXERCISES

Simplify. Give the answers in scientific notation.

1. $3,000,000 \times 2,000$

2. $4,000,000 \times 1,400,000$

3. $23,000,000 \times 170,000$

4. $4 \times 10^8 \times 2 \times 10^7$

5. $2.3 \times 10^6 \times 3 \times 10^4$

6. $3.6 \times 10^9 \times 1.23 \times 10^8$

7. $6,000,000 \times 9,000,000$

8. $8 \times 10^5 \times 6 \times 10^4$

9. $2.4 \times 10^8 \times 5.3 \times 10^9$

10. $6.1 \times 10^8 \times 2.45 \times 10^9$

11. $\frac{8,000,000,000}{4,000}$

12. $\frac{9,000,000}{30,000}$

13. $\frac{6,900,000}{3,000}$

14. $\frac{8 \times 10^6}{4 \times 10^2}$

15. $\frac{9 \times 10^{14}}{3 \times 10^2}$

16. $\frac{3.6 \times 10^8}{3 \times 10^4}$

17. $\frac{16.8 \times 10^8}{4 \times 10^3}$

18. $\frac{4.2 \times 10^9}{1.4 \times 10^7}$

19. $\frac{7.2 \times 10^9}{1.2 \times 10^4}$

Solve these problems.

20. Earth is your space ship. It travels about 600,000,000 miles a year. How far in space has a 14 year old student traveled?

★ **21.** The sun is about 93,000,000 miles from the earth. The moon is about 240,000 miles from the earth. The distance to the sun is about how many times the distance to the moon?

CHAPTER REVIEW

Compute.

1. $4.2 + 36 + 9.8$ [52]

2. $47 - 2.8$ [54]

3. $36 + 1.7 + .246$ [52]

4. $3.8 - .026$ [54]

5. [56] $\begin{array}{r} .067 \\ \times\ .8 \\ \hline \end{array}$

6. [56] $\begin{array}{r} 2.43 \\ \times\ 3.7 \\ \hline \end{array}$

7. $.2\overline{).08}$ [60]

8. $2.8\overline{).168}$ [60]

Give the quotients to the nearest tenth. [62]

9. $8\overline{)3.7}$

10. $.8\overline{).3}$

11. [64] Within what interval must the exact length of a line segment fall if it measures 5″ to the nearest inch?

12. [64] Which is more precise, 3″ or $1\frac{1}{2}$″?

13. [66] What is the greatest possible error of the measurement 1.9″?

14. [68] Within what interval does the sum of 23.4″ and 16.8″ fall?

15. [70] Add the measurements: 4.2″ + 3.48″ + 4.234″.

16. [70] Subtract the measurements: 6.24″ − 2.8″

Give the number of significant digits. [72]

17. 35′

18. .008 cm

19. .400″

20. [74] Two measurements are 21.24″ and 8.4″. Find the product to the correct number of significant digits.

Give the answers in scientific notation. [78]

21. $8{,}000{,}000{,}000 \times 4{,}000$

22. $\frac{22{,}000{,}000{,}000}{11{,}000{,}000}$

CHAPTER TEST

Compute.

1. $8.3 + 2.7 + 6.8$

2. $3.4 - 1.6$

3. $.35 + 1.6 + .416$

4. $2.3 - .056$

5. $\begin{array}{r} .056 \\ \times\ .9 \\ \hline \end{array}$

6. $\begin{array}{r} 1.34 \\ \times\ 2.6 \\ \hline \end{array}$

7. $31\overline{).093}$

8. $2.6\overline{).234}$

Give the quotients to the nearest tenth.

9. $.3\overline{)5.6}$

10. $.7\overline{).2}$

11. Within what interval must the actual length of a line segment fall if it measures 3″ to the nearest inch?

12. Which is more precise, $5\frac{1}{4}''$ or $2\frac{1}{2}''$?

13. What is the greatest possible error of the measurement 3.2″?

14. Within what interval does the sum of 14.6″ and 14.9″ fall?

15. Add the measurements: $4.1'' + 2.36'' + 5.056''$.

16. Subtract the measurements: $5.62'' - 1.7''$.

Give the number of significant digits.

17. 64″

18. .004 cm

19. .300″

20. Multiply the measurements $14.37'' \times 6.4''$. Give the product to the correct number of significant digits.

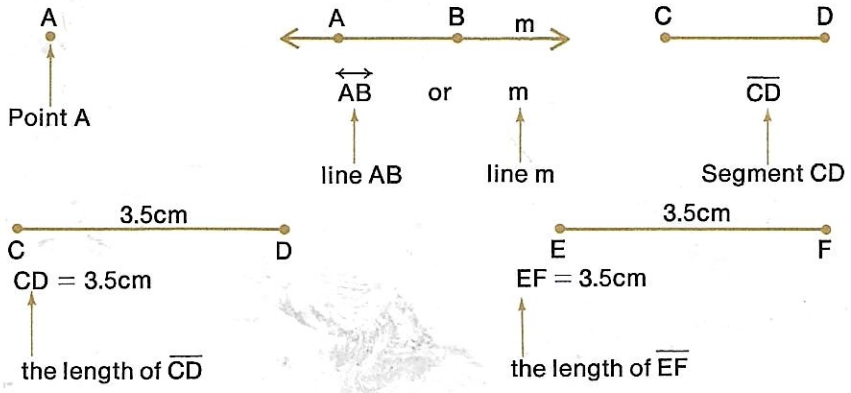
Give the answers in scientific notation.

21. $7{,}000{,}000 \times 4{,}000{,}000$

22. $\frac{7{,}200{,}000{,}000}{1{,}200}$

4 GEOMETRY

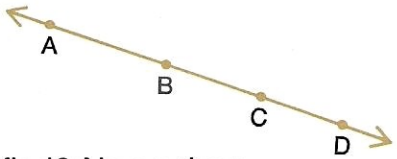
POINTS, LINES, AND SEGMENTS



$\overline{CD}$ and $\overline{EF}$ have the same length. $\overline{CD} \cong \overline{EF}$.

read: **is congruent to**

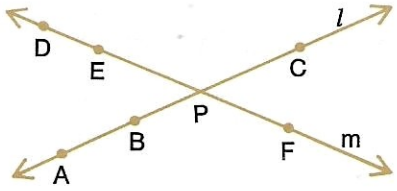
1. Consider this line.



a. How many points are named? List them.

b. How many segments can you find? Name them.

2. a. Points *A, B, P,* and *C* are **collinear** points because they are on the same line. Name another set of collinear points.



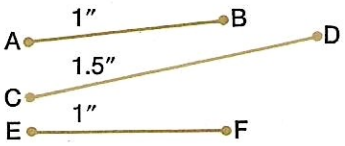
b. What point is common to ℓ and m?

3. a. Point *P* separates $\overleftrightarrow{AB}$ into two **half-lines** $\overrightarrow{PA}$ and $\overrightarrow{PB}$. What does the circle mean?

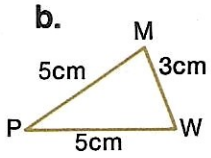
b. Name two half-lines for point *R*.

4. Which segments are congruent? Write sentences using $\cong$.

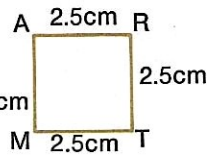
a.



b.

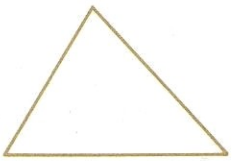


c.

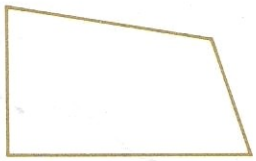


5. A **polygon** is a simple closed curve which is the union of line segments. Name these polygons.

a.



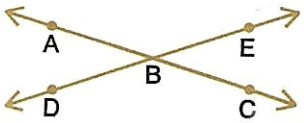
b.



c.

EXERCISES

Consider these lines.



1. Name two sets of collinear points.

2. Name two half-lines of AC and of DE for point B.

3. What is $\overleftrightarrow{AC} \cap \overleftrightarrow{DE}$?

Consider this polygon.

4. How many different segments can you find? Name them.

5. $\overline{FC}$ is a **diagonal**. Name the other diagonals shown.

6. What is $\overline{AP} \cup \overline{PD}$? **7.** What is $\overline{AP} \cap \overline{PD}$? **8.** Find $\overline{AF} \cap \overline{AB}$.

Which pairs of segments are congruent? Use your ruler.

9.

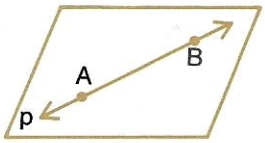
10.

11.

PLANES, LINES, AND POINTS

Let's experiment with planes. Use a piece of thick paper as a model of a part of a plane.

1. a. Place the paper on your desk. Label it plane p.



b. Mark points A and B on the paper. Draw $\overleftrightarrow{AB}$.

c. Will any point of $\overleftrightarrow{AB}$ not be on the plane?

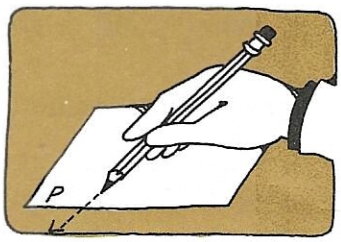
If two points of a line are on a plane, then all points of the line are on the plane.

2. a. Think of the floor of your room as a plane. Think of the ceiling of your room as another plane. What kind of planes are they?

b. Will any line in the ceiling intersect the plane of the floor?

A line and a plane need not intersect.

3. Use a pencil as a physical model of a line. (We are pretending that the pencil is unending.) Hold the pencil as shown in the diagram. The line is not on the plane. At how many points can the pencil intersect the plane?



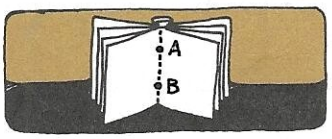
If a plane and a line, which is not on the plane, intersect, then the intersection is a point.

4. a. Fold a piece of paper. What geometric figure is suggested by the fold?

b. What can you say about the intersection of two planes?

If two planes intersect, then the intersection is a line.

5. Think of the pages of your math book as planes. How many planes contain $\overleftrightarrow{AB}$?



An infinite set of planes can contain one line.

6. A side wall, front wall, and ceiling meet. What is their intersection?

Three planes can meet in one point.

7. **a.** Hold one pencil under a piece of cardboard. Does the cardboard tilt in all directions?

   b. Hold two pencils under the cardboard. Does the cardboard tilt about the points? How many planes can contain two points?

   c. Hold three pencils under the cardboard. Try to tilt the cardboard. What do you find?

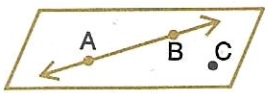
Three **noncollinear** points determine exactly one plane.

EXERCISES

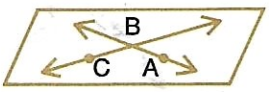
1. If two lines intersect, what is the intersection?

2. How many points determine a line? a plane?

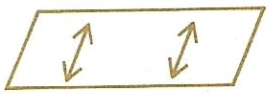
3. How many planes will contain a line and a point not on the line?



4. How many planes will contain two intersecting lines?



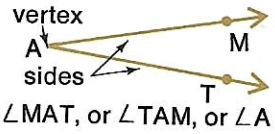
5. How many planes will contain two parallel lines?



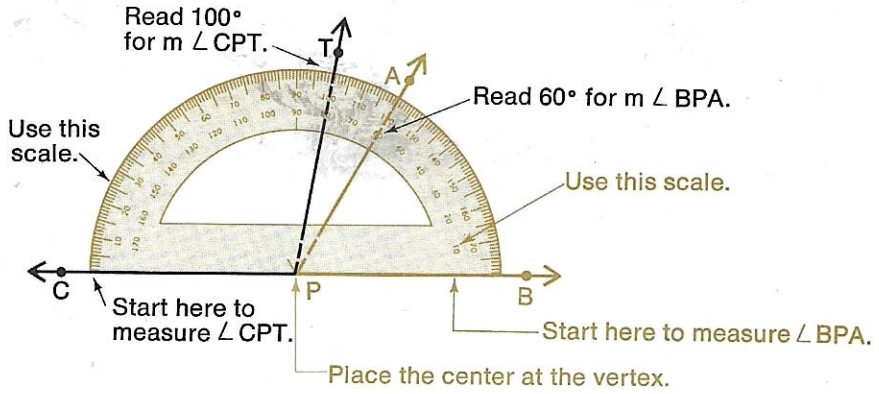
6. How many planes will contain a triangle? a quadrilateral?

ANGLES

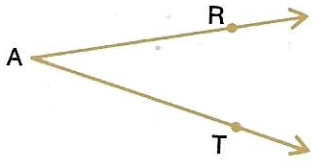
An **angle** is the union of two rays with a common endpoint. $\overrightarrow{AM}$ (read: ray *AM*) and $\overrightarrow{AT}$ are the sides of $\angle A$.



We measure angles with a **protractor**.



1. Consider this angle.

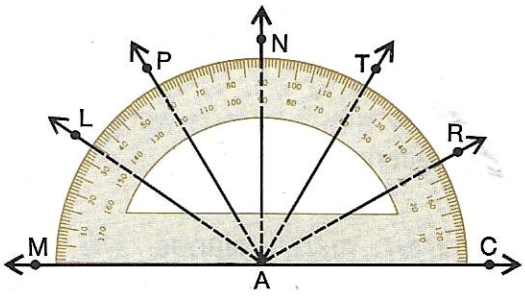


   a. Name the vertex.

   b. Name the sides.

   c. Name the angle in 3 ways.

2. Give the measures.



   a. $\angle CAR$ **b.** $\angle CAN$

   c. $\angle MAP$ **d.** $\angle MAN$

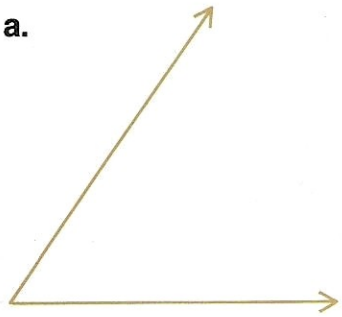
   e. $\angle MAT$ **f.** $\angle CAP$

   g. $\angle CAT$ **h.** $\angle MAL$

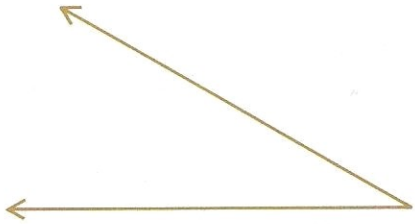
3. If two angles have the same measure, they are congruent. Which angles in Item 2 are congruent?

4. Measure these angles.

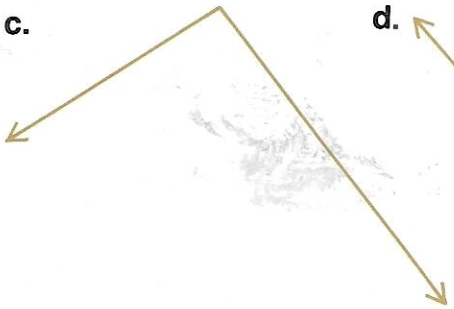
a.



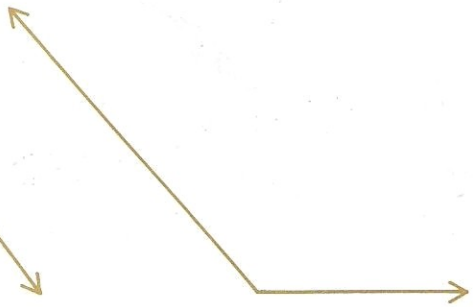
b.



c.



d.



5. Draw angles with these measures.

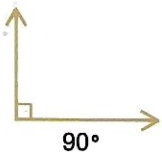
a. 40° **b.** 90° **c.** 160° **d.** 35° **e.** 73°

6. We classify angles according to their measures.

Acute Angle

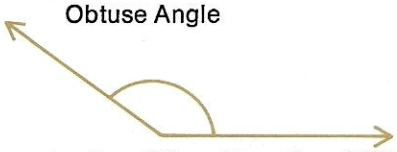
less than 90°

Right Angle



90°

Obtuse Angle



greater than 90° and less than 180°

Straight Angle

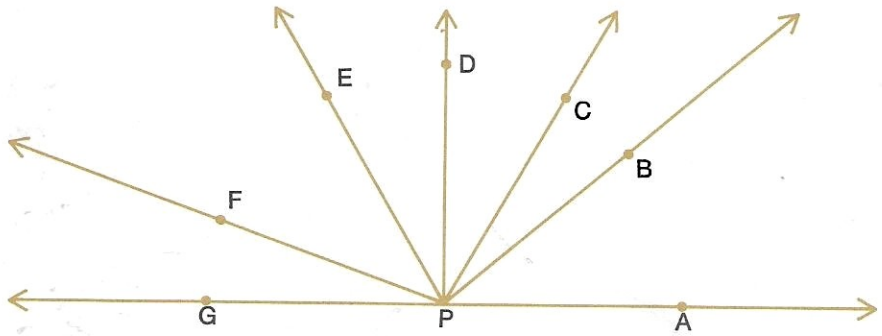
180°

What kinds of angles are represented by these measures?

a. 139° **b.** 30° **c.** 90° **d.** 89° **e.** 180°

EXERCISES

Measure the angles.



1. $\angle APB$ **2.** $\angle APC$ **3.** $\angle APD$ **4.** $\angle APE$

5. $\angle APF$ **6.** $\angle GPF$ **7.** $\angle GPE$ **8.** $\angle GPD$

9. $\angle GPC$ **10.** $\angle GPB$ **11.** $\angle BPC$ **12.** $\angle BPD$

13. $\angle CPD$ **14.** $\angle CPE$ **15.** $\angle FPE$ **16.** $\angle APG$

Consider your answers to Exercises 1–16.

17. Classify the angles as acute, right, obtuse, or straight.

18. Which pairs of angles are congruent?

Draw angles with these measures.

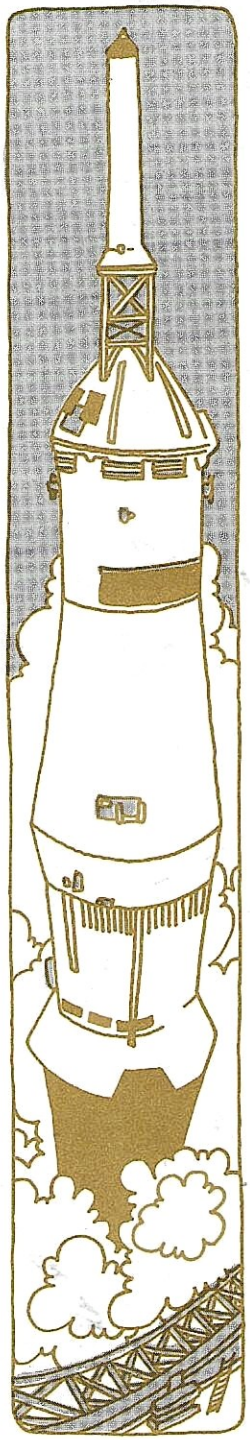
19. 50° **20.** 120° **21.** 78° **22.** 113° **23.** 45°

BRAINTEASERS

1. A man fell into a 30-foot well. Each day he climbed 3 feet, but slid back 2 feet at night. How long did it take him to get out?

2. Copy the 9 equally, spaced dots on a piece of paper. Put your pencil point on one of the dots. Without lifting your pencil from the paper, connect the dots with only 4 line segments.

REACH FOR THE SKY



1. John Glenn was the first American to orbit the earth. His flight lasted 4 hours 55 minutes. Armstrong, Aldrin, and Collins were the first Americans to go to the moon. The moon flight lasted 195 hours 18 minutes. Find the difference in flight times.

2. In 1750, the altitude record for rockets was .71 miles. In 1931, the record was 1.24 miles. Find the difference in feet, in the records.

3. Mars has two moons, Phobos and Deimos. Phobos takes about .32 days to go around Mars; Deimos takes about 1.26 days. About how many times will Phobos go around Mars while Deimos makes one revolution?

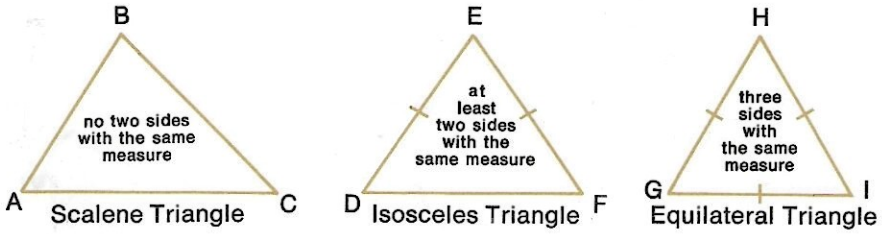
4. One astronomical unit is 93 million miles, the average distance of the earth from the sun. Mercury is .387 astronomical units from the sun. How many miles is Mercury from the sun?

5. At an altitude of 100 miles, a spacecraft orbits the earth in 1.43 hours. How many minutes does it take to orbit the earth?

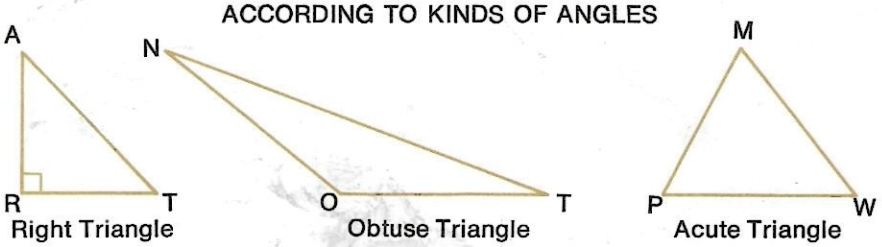
6. One problem in space travel is weightlessness. It has been suggested that a space station, something like a tire, be built so that it would rotate and we could walk as on Earth. If the station makes .057 rotations per second, how many does it make per hour?

CLASSIFYING TRIANGLES

ACCORDING TO THE MEASURES OF THE SIDES

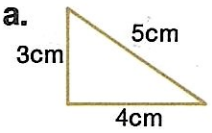


ACCORDING TO KINDS OF ANGLES

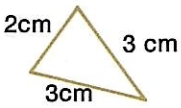


1. Which triangles are equilateral? isosceles? scalene?

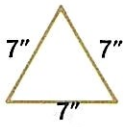
a.



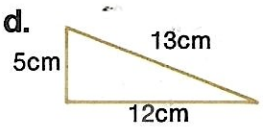
b.



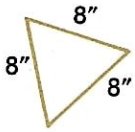
c.



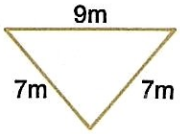
d.



e.

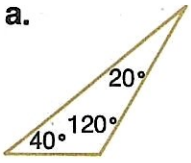


f.

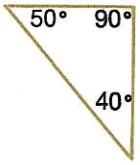


2. Which triangles are acute? right? obtuse?

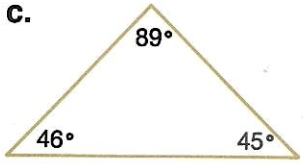
a.



b.



c.



3. True or false?

a. All isosceles triangles are also equilateral triangles.

b. All equilateral triangles are also isosceles triangles.

4. **a.** Draw an angle of 90°. Use your protractor.

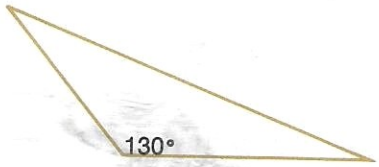
   b. Mark off 1″ on each ray.

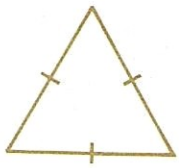
   c. Connect the endpoints.

   d. What kind of triangle is formed?

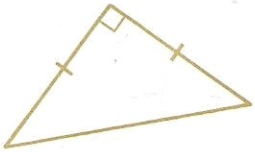
EXERCISES

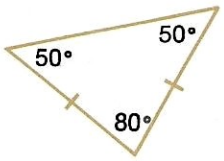
Classify these triangles according to the measures of their angles and sides.

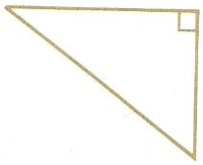
1. 

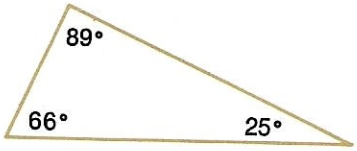
2. 

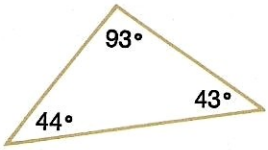
3.

4. 

5. 

6. 

7. 

8. 

Draw an example of each triangle.

9. Obtuse isosceles

10. Acute isosceles

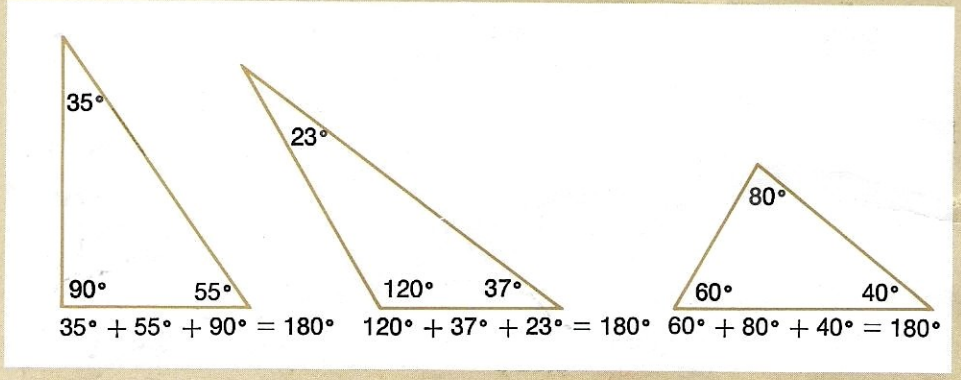
11. Right scalene

12. Obtuse scalene

13. Acute scalene

14. Equilateral

SUM OF THE MEASURES OF THE ANGLES OF A TRIANGLE



$35° + 55° + 90° = 180°$ $120° + 37° + 23° = 180°$ $60° + 80° + 40° = 180°$

1. **a.** Draw a large triangle.

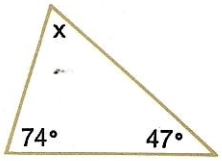
b. Measure each angle.

c. What is the sum of the measures of the angles?

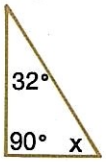
The sum of the measures of the angles of any triangle is 180°.

2. Find the measures of the angles labeled x.

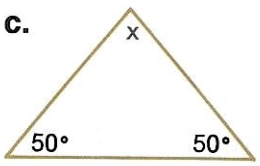
a.



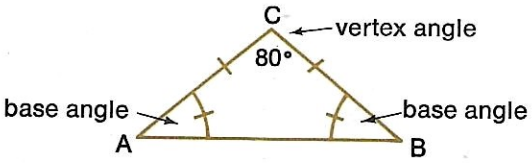
b.



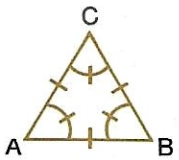
c.



3. The **base angles** of an isosceles triangle are congruent. Find the measure of each base angle.



4. All angles of an equilateral triangle have the same measure. Find the measure of each angle of $\triangle ABC$.



5. Why can't a triangle have more than one right angle?

6. Why can't a triangle have more than one obtuse angle?

EXERCISES

Complete this chart.

	$m\angle A$	$m\angle B$	$m\angle C$	*Kind of* $\triangle$
	40°	40°	100°	Obtuse
1.	72°	68°		Acute
2.		40°	90°	
3.	27°	110°		
4.	72°		70°	
5.	123°		31°	
6.	30°		85°	
7.	56°		54°	
8.	45°	90°		
9.	70°		70°	
10.		32°	32°	
11.	150°		17°	
12.	75°	43°		

Solve these problems.

13. In $\triangle ABC$, $m\angle A = 67°$ and $m\angle B = 33°$. What is $m\angle C$?

14. The measure of the vertex angle of an isosceles triangle is 70°. What is the measure of each base angle?

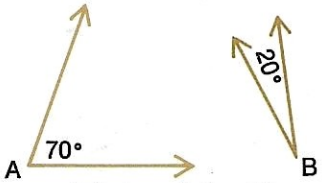
15. One acute angle of a right triangle measures 40°. What is the measure of the other acute angle?

16. What is the sum of the measures of the two acute angles of a right triangle?

★**17.** In $\triangle ABC$, $m\angle B = 3 \cdot m\angle A$ and $m\angle C = 5 \cdot m\angle A$. What is the measure of each angle?

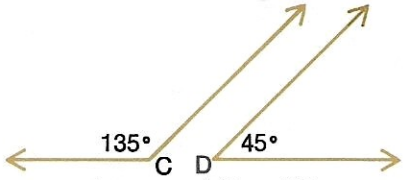
PAIRS OF ANGLES

Complementary Angles



$m\angle A + m\angle B = 90°$

Supplementary Angles



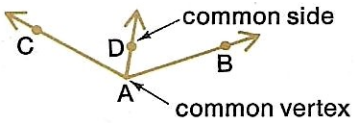
$m\angle C + m\angle D = 180°$

1. Complementary or supplementary?

a. 30°, 60° **b.** 72°, 18° **c.** 100°, 80°

d. 110°, 70° **e.** 90°, 90° **f.** 24°, 66°

2. $\angle DAB$ and $\angle DAC$ are **adjacent angles**. $\angle CAB$ and $\angle CAD$ are not adjacent angles. True or false?

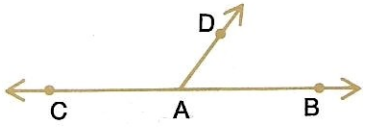


a. $\angle CUP$ and $\angle CUT$ are adjacent.

b. $\angle CUP$ and $\angle PUG$ are adjacent.

c. $\angle TUG$ and $\angle CUP$ are adjacent.

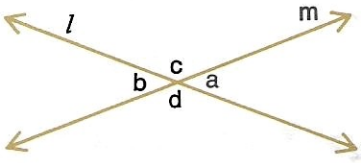
3. Consider this figure.



a. What kind of angles are $\angle DAC$ and $\angle DAB$?

b. What kind of angle is $\angle CAB$? So, $\angle DAC$ and $\angle DAB$ are **supplementary angles.**

4. Lines ℓ and m intersect. $\angle a$ and $\angle b$ are **vertical angles**.



a. Name another pair of vertical angles.

b. Measure $\angle a$ and $\angle b$. **c.** Measure $\angle c$ and $\angle d$.

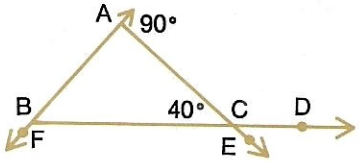
Vertical angles are congruent.

5. Find the measures. Do not use your protractor.

a. $\angle BAC$ **b.** $\angle ACD$

c. $\angle DCE$ **d.** $\angle ABC$

e. $\angle BCE$ **f.** $\angle FBC$



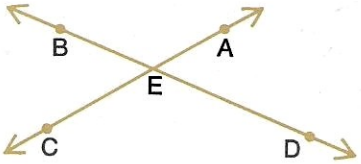
EXERCISES

Complementary or supplementary?

1. 24°, 156° **2.** 80°, 10° **3.** 90°, 90° **4.** 40°, 140°

5. 45°, 45° **6.** 12°, 78° **7.** 137°, 43° **8.** 164°, 16°

Consider this figure.



9. Name pairs of adjacent angles.

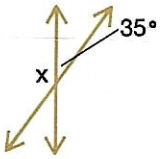
10. Name pairs of vertical angles.

11. Which angles are congruent?

12. Which angles are supplementary?

Find $m\angle x$ in each picture. Do not use your protractor.

13.



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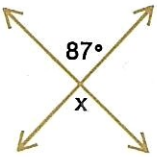

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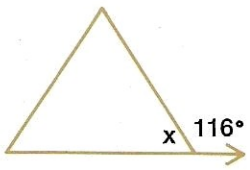

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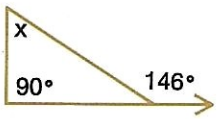
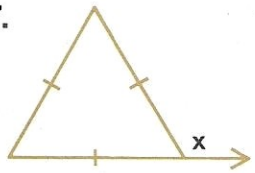
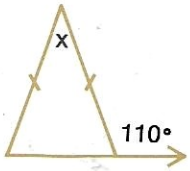


17.



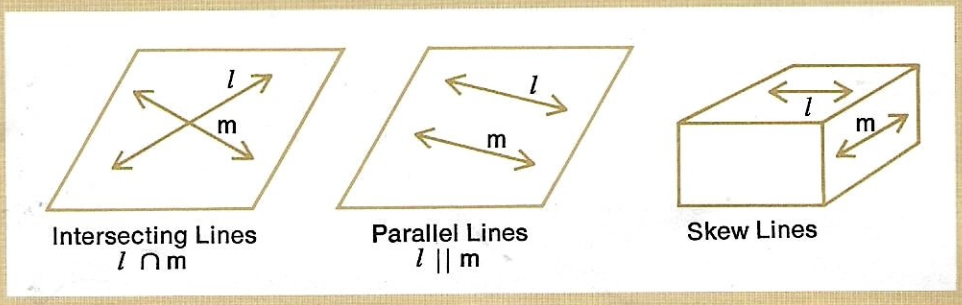
18.



Solve this problem.

★ **19.** $\angle C$ and $\angle D$ are supplementary angles. $m\angle C = 2 \cdot m\angle D$. What are the measures of $\angle C$ and $\angle D$?

PARALLEL LINES AND PAIRS OF ANGLES

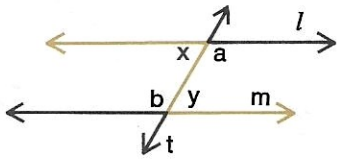


Intersecting Lines
$l \cap m$

Parallel Lines
$l \parallel m$

Skew Lines

1. In this figure, $\ell \parallel m$ and t is a **transversal**. $\angle x$ and $\angle y$ are **alternate interior angles**. Name another pair of alternate interior angles.



2. **a.** Draw a figure like the one in Item 1.

b. Measure $\angle x$ and $\angle y$. What do you find?

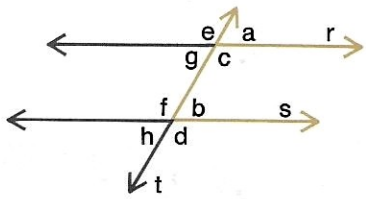
c. Without measuring, find $m\angle a$. (Hint: $m\angle x + m\angle a = 180°$.)

d. Measure $\angle b$. Complete: $m\angle a$ ___ $m\angle b$.

e. What is true of each pair of alternate interior angles?

If two parallel lines are cut by a transversal, the alternate interior angles are congruent.

3. In this figure, $r \parallel s$ and t is a transversal. $\angle a$ and $\angle b$ are **corresponding angles**. Name 3 more pairs of corresponding angles.



4. **a.** Draw a picture like the one in Item 3.

b. Measure $\angle a$ and $\angle b$. What do you find?

c. Without measuring, find $m\angle c$ and $m\angle d$.

d. Without measuring, find $m\angle e$ and $m\angle f$. (Hint: $\angle c$ and $\angle e$, and $\angle d$ and $\angle f$ are vertical angles.)

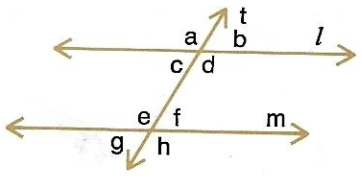
e. Find $m\angle g$ and $m\angle h$.

f. What is true of each pair of corresponding angles?

If two parallel lines are cut by a transversal, the corresponding angles are congruent.

EXERCISES

In this figure, $\ell \parallel m$ and t is a transversal.

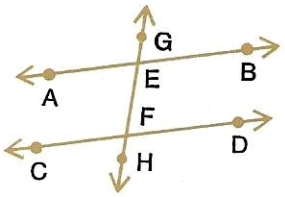


1. Name the pairs of alternate interior angles.

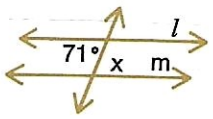
2. Name the pairs of corresponding angles.

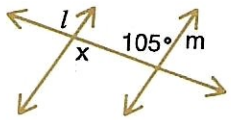
In this figure, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{GH}$ is a transversal. True or false?

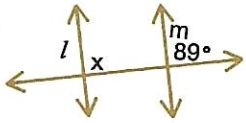
3. $\angle AEF \cong \angle EFD$
4. $\angle GEA \cong \angle EFC$
5. $\angle BEF \cong \angle EFD$
6. $\angle GEB \cong \angle AEF$
7. $\angle BEF \cong \angle DFH$
8. $\angle BEF \cong \angle CFH$

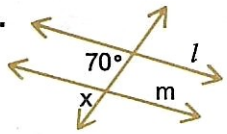


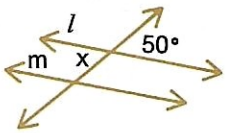
In each case, $\ell \parallel m$. Find $m\angle x$. Do not use a protractor.

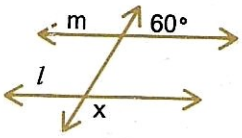
9. 

10. 

11. 

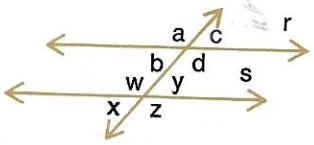
12. 

13. 

14. 

In this figure, $r \parallel s$ and $m\angle a = 110°$.

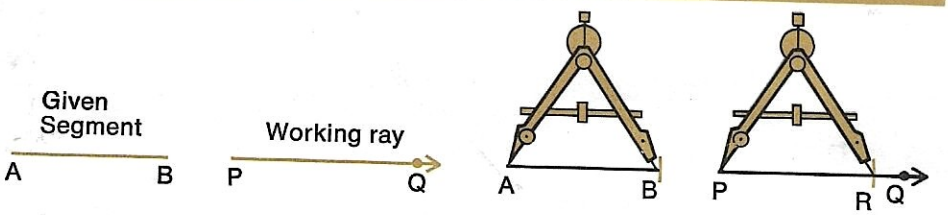
15. Find the measure of each of the other angles.



BASIC CONSTRUCTIONS

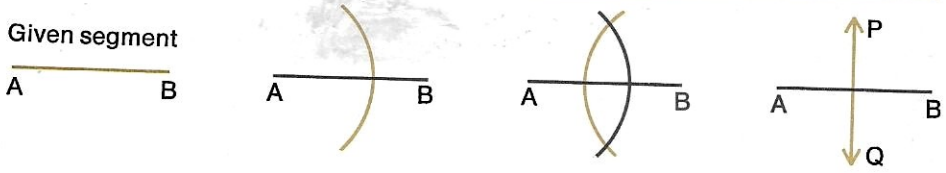
Here are some constructions with which you should be familiar.

COPYING A SEGMENT



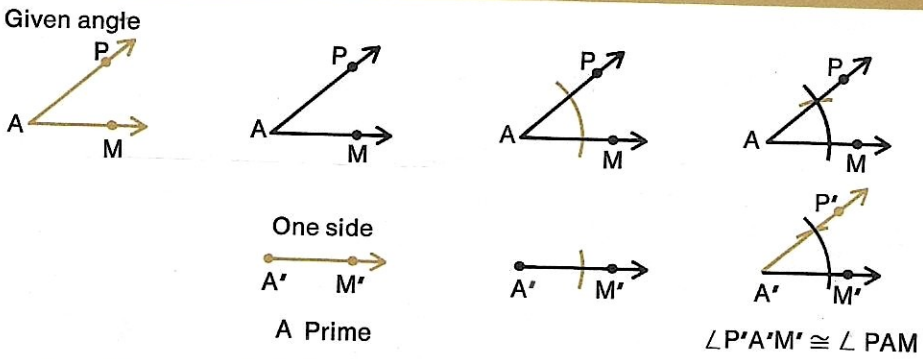
$\overline{PR} \cong \overline{AB}$

BISECTING A SEGMENT



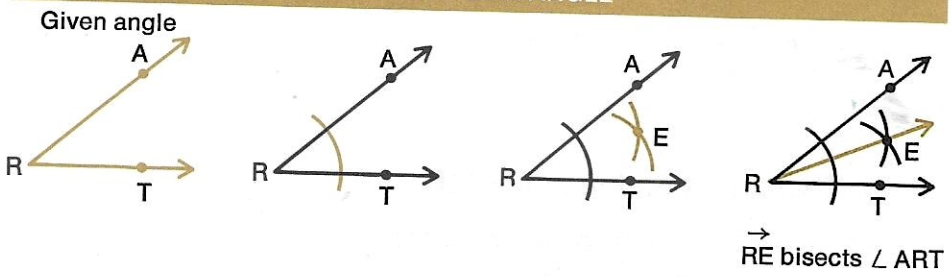
$\overleftrightarrow{PQ}$ bisects $\overline{AB}$

COPYING AN ANGLE



$\angle P'A'M' \cong \angle PAM$

BISECTING AN ANGLE



$\overrightarrow{RE}$ bisects $\angle ART$

EXERCISES

1. Draw a segment of any length you wish. Construct a segment congruent to it.

2. Draw a segment. Bisect it.

3. Draw an acute angle. Construct an angle congruent to it.

4. Draw an obtuse angle. Construct an angle congruent to it.

5. Draw an acute angle, a right angle, and an obtuse angle. Bisect each angle.

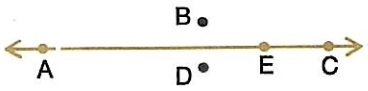
1. Give 2 names for the line.

2. How many lines can contain one point?

3. How many lines can contain two points?

4. What is the greatest number of points in which two lines intersect?

5. Which points are collinear?



6. What is $\overrightarrow{OA} \cap \overrightarrow{OB}$?

7. What does $\overline{MN}$ mean?

8. What does $\overrightarrow{MN}$ mean?

9. An angle is named $\angle ABC$. Name it in two other ways.

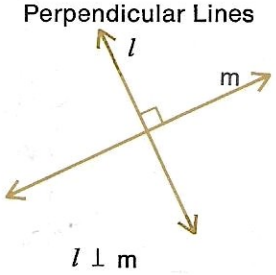
10. Name four kinds of polygons.

11. Lines m and n are parallel. What is $m \cap n$?

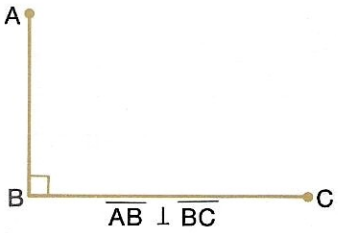
12. How many sides does a quadrilateral have?

CONSTRUCTING PERPENDICULAR LINES

Perpendicular Lines

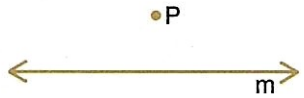


Perpendicular segments



Lines or segments of lines which intersect at right angles are perpendicular.

1. a. Draw a picture like this.



b. Through P, draw lines which intersect m. How many such lines are there?

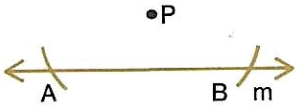
c. How many of the lines are perpendicular to m? *1*

Through a point P not on line m, there is exactly one line perpendicular to m.

2. Point P is not on line m. From P, let's construct the line perpendicular to m.

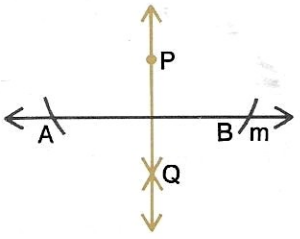
a. Draw line m and mark point P.

b. Place the foot of the compass at P, and draw arcs which cut m at A and B.



c. Place the foot of the compass at A, and draw an arc. Don't change the compass opening. Place the foot of the compass at B, and draw another arc.

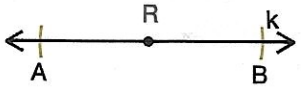
d. Call the point of intersection of the arcs Q. Draw $\overleftrightarrow{PQ}$. $\overleftrightarrow{PQ} \perp m$.



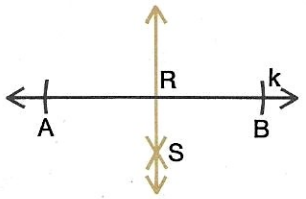
3. Point R is on line k. At R, let's construct the line perpendicular to k.

a. Draw line k and mark point R.

b. Place the foot of the compass at R, and draw arcs which cut k at A and B.



c. Set the compass opening greater than the length of $\overline{AR}$. Place the foot of the compass at A, and draw an arc. Don't change the compass opening. Place the foot of the compass at B, and draw an arc.



d. Call the point of intersection of the arcs S. Draw $\overleftrightarrow{RS}$. $\overleftrightarrow{RS} \perp k$.

EXERCISES

Copy the drawings. From P, construct the line perpendicular to line n.

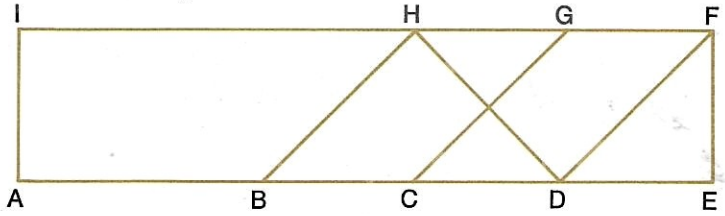
1.

2.

3.

4.

Consider the drawing.

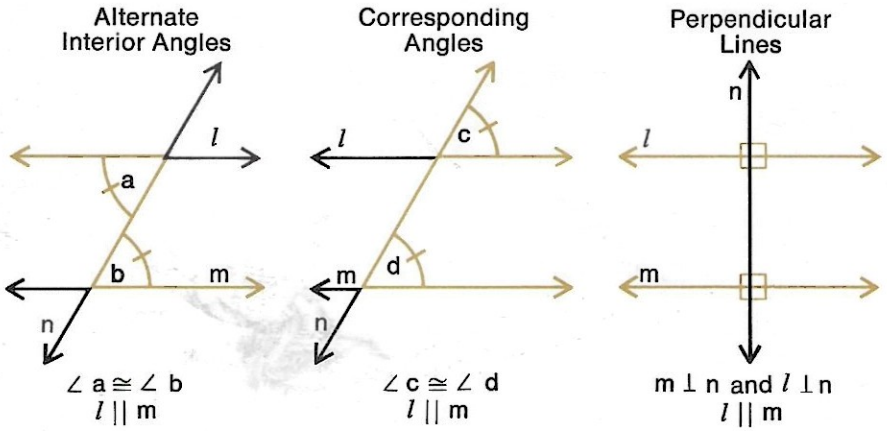


5. Which segments look perpendicular?

6. Which segments look parallel?

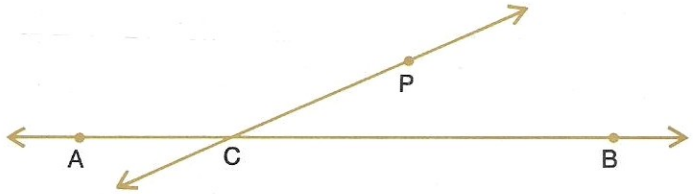
CONSTRUCTING PARALLEL LINES

The pictures suggest three ways to construct a pair of parallel lines.

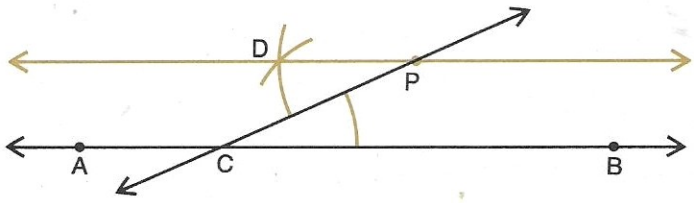


1. Point P is not on $\overleftrightarrow{AB}$. Through P, let's construct a line parallel to $\overleftrightarrow{AB}$. We shall use alternate interior angles.

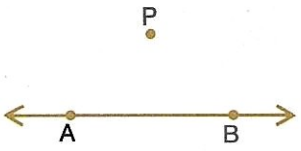
a. Draw $\overleftrightarrow{AB}$ and mark point P. Through P, draw a line which intersects $\overleftrightarrow{AB}$ at C.



b. Use P as a vertex. Copy $\angle PCB$ on the opposite side of $\overleftrightarrow{CP}$. Mark point D. Draw $\overleftrightarrow{DP}$. $\overleftrightarrow{DP} \parallel \overleftrightarrow{AB}$.



2. a. Copy this picture.



b. Construct $\overleftrightarrow{PQ} \parallel \overleftrightarrow{AB}$. Use corresponding angles.

3. a. Draw $\overleftrightarrow{AB}$. Mark point P not on $\overleftrightarrow{AB}$.

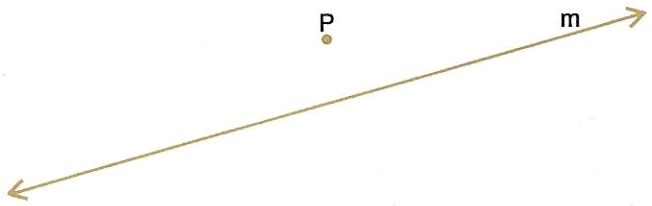
b. From P, construct a line perpendicular to $\overleftrightarrow{AB}$. Call the line that you constructed $\overleftrightarrow{PQ}$.

c. From P, construct a line perpendicular to $\overleftrightarrow{PQ}$. Mark a point D on the line that you constructed.

d. How is $\overleftrightarrow{PD}$ related to $\overleftrightarrow{AB}$?

EXERCISES

1. Copy this picture. Through P, construct a line parallel to m. Use the idea of alternate interior angles.



2. Copy this picture. Through X, construct a line parallel to p. Use the idea of corresponding angles.

3. Copy this picture. Through M, construct a line parallel to w. Use the idea of perpendicular lines.

CONSTRUCTING SPECIAL ANGLES

We can construct some angles with just a straightedge and compass.

1. Let's construct an angle whose measure is 45°.

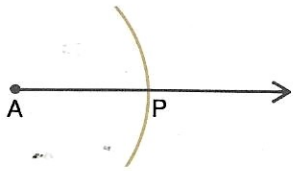
a. Construct a pair of perpendicular lines. What is the measure of each angle?

b. Bisect one of the angles.

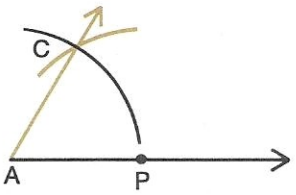
2. Let's construct an angle whose measure is 60°.

a. Draw a working ray.

b. With A as a center, draw an arc cutting the ray at P.



c. Don't change the compass opening. With P as a center, draw an arc cutting the other arc at C. Draw $\overrightarrow{AC}$.



d. $m\angle CAP = 60°$. Check with your protractor.

3. Let's construct an angle whose measure is 30°.

a. Construct an angle whose measure is 60°.

b. Bisect the angle.

EXERCISES

Construct angles with these measures.

1. 90° **2.** 60° **3.** 45°

4. 30° **5.** 15° ★**6.** 120°

Add.

1. $\begin{array}{r} .7 \\ .6 \\ +.9 \\ \hline \end{array}$ **2.** $\begin{array}{r} .04 \\ .37 \\ +.65 \\ \hline \end{array}$ **3.** $\begin{array}{r} 3.4 \\ 6.8 \\ +2.9 \\ \hline \end{array}$

4. $5.046 + 7.659 + 8.368 + 3$

5. $.356 + .204 + .619$

6. $.0014 + .0379 + .6842$

7. $6.57 + 6.9 + 7.843$

8. $8.92 + 7.246 + 2.4361$

Subtract.

9. $\begin{array}{r} .8 \\ -.4 \\ \hline \end{array}$ **10.** $\begin{array}{r} .36 \\ -.14 \\ \hline \end{array}$ **11.** $\begin{array}{r} .040 \\ -.012 \\ \hline \end{array}$ **12.** $\begin{array}{r} .3416 \\ -.2413 \\ \hline \end{array}$

13. $12.4 - 4.6$

14. $3.59 - 1.64$

15. $4.24 - 3.1823$

Multiply.

16. $\begin{array}{r} .3 \\ \times .6 \\ \hline \end{array}$ **17.** $\begin{array}{r} .04 \\ \times 7 \\ \hline \end{array}$ **18.** $\begin{array}{r} .031 \\ \times 9 \\ \hline \end{array}$ **19.** $\begin{array}{r} .213 \\ \times 8 \\ \hline \end{array}$

20. $.4 \times .3$

21. $.04 \times .6$

22. $.005 \times .7$

23. $.0136 \times .6$

24. $1.4 \times .6$

25. 2.7×3.4

26. $3.05 \times .16$

27. 4.08×100

28. 4.231×2.16

Divide.

29. $2\overline{)2.4}$

30. $6\overline{).024}$

31. $8\overline{).0240}$

32. $3\overline{)1.5}$

33. $5\overline{).025}$

34. $4\overline{)24.36}$

35. $.2\overline{).8}$

36. $.4\overline{)1.6}$

37. $.5\overline{)25}$

38. $.4\overline{)2.36}$

39. $.06\overline{)360}$

40. $.09\overline{)21.24}$

Let $A = \{\text{Bob, John, Jim}\}$ and $B = \{\text{Carol, Sue, Ann, Amy}\}$.

41. What is $A \cup B$?

42. What is $A \cap B$?

CHAPTER REVIEW

1. The sides of $\triangle ABC$ measure 2″, 1″, and 2″. What kind of [90] triangle is $\triangle ABC$?

2. The angles of $\triangle RST$ measure 50°, 60°, and 70°. What kind [90] of triangle is $\triangle RST$?

3. In triangle HIJ, $m\angle H = 81°$ and $m\angle I = 38°$. Find $m\angle J$. [92]

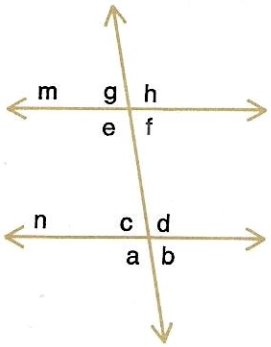
4. $\angle A$ and $\angle B$ are complementary angles. $m\angle A = 40°$. What [94] is $m\angle B$?

5. $\angle C$ and $\angle D$ are supplementary angles. $m\angle C = 41°$. What is [94] $m\angle D$?

In the picture, $m \parallel n$. [96]

6. Name a pair of alternate interior angles.

7. Name a pair of corresponding angles.



8. $m\angle f = 80°$. Find $m\angle c$.

9. Find $m\angle b$.

10. True or false? $\angle d \cong \angle f$.

11. True or false? $\angle g$ and $\angle h$ are vertical angles.

12. Draw $\overleftrightarrow{AB}$. Mark a point P not on $\overleftrightarrow{AB}$. Through P, construct [100] a line perpendicular to $\overleftrightarrow{AB}$.

13. Draw $\overleftrightarrow{RT}$. Mark a point D on $\overleftrightarrow{RT}$. At D, construct a line per- [100] pendicular to $\overleftrightarrow{RT}$.

14. Draw $\overleftrightarrow{EF}$. Mark a point R not on $\overleftrightarrow{EF}$. Through R, construct [102] a line parallel to $\overleftrightarrow{EF}$.

15. Construct an angle of 45°. [104]

CHAPTER TEST

1. The sides of $\triangle QRS$ measure 3″, 4″, and 6″. What kind of triangle is $\triangle QRS$?

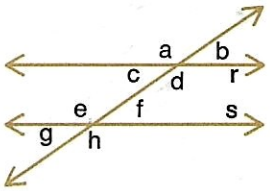
2. The angles of $\triangle DEF$ measure 50°, 110°, and 20°. What kind of triangle is $\triangle DEF$?

3. In triangle DEF, $m\angle D = 91°$ and $m\angle E = 88°$. Find $m\angle F$.

4. $\angle A$ and $\angle B$ are complementary angles. $m\angle A = 63°$. What is $m\angle B$?

5. $\angle C$ and $\angle D$ are supplementary angles. $m\angle C = 84°$. What is $m\angle D$?

In the picture, $r \parallel s$.



6. Name a pair of alternate interior angles.

7. Name a pair of corresponding angles.

8. $m\angle c = 35°$. Find $m\angle f$.

9. Find $m\angle g$.

10. True or false? $\angle e \cong \angle a$.

11. True or false? $\angle e$ and $\angle f$ are vertical angles.

12. Draw $\overleftrightarrow{AB}$. Mark point P not on $\overleftrightarrow{AB}$. Through P, construct a line perpendicular to $\overleftrightarrow{AB}$.

13. Draw $\overleftrightarrow{PW}$. Mark a point M on $\overleftrightarrow{PW}$. At M, construct a line perpendicular to $\overleftrightarrow{PW}$.

14. Draw $\overleftrightarrow{CD}$. Mark point Q not on $\overleftrightarrow{CD}$. Through Q, construct a line parallel to $\overleftrightarrow{CD}$.

15. Construct an angle of 30°.

TABLE OF MEASURES

Length

1 foot (ft) = 12 inches (in.)
1 yard (yd) = 3 feet
1 mile (mi) = 5,280 feet
1 mile = 1,760 yards

1 centimeter (cm) = 10 millimeters (mm)
1 decimeter (dm) = 10 centimeters
1 meter (m) = 10 decimeters
1 dekameter (dam) = 10 meters
1 hectometer (hm) = 100 meters
1 kilometer (km) = 1,000 meters

1 inch = 2.54 centimeters
1 centimeter ≐ .39 inches
1 meter ≐ 39 inches
1 kilometer ≐ .62 miles

Liquid

1 fluid ounce (fl oz) = 2 tablespoons (tbs)
1 cup = 8 fluid ounces
1 pint (pt) = 2 cups
1 quart (qt) = 2 pints
1 gallon (gal) = 4 quarts

1 liter (ℓ) = 1,000 cubic centimeters (cc)
1 liter ≐ 1 quart

Weight

1 pound (lb) = 16 ounces (oz)
1 ton (T) = 2,000 pounds

1 gram (g) = 1,000 milligrams (mg)
1 kilogram (kg) = 1,000 grams
1 kilogram ≐ 2 pounds

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